Inapproximability results for graph convexity parameters

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This is a joint work with Érika Coelho (UFG, Goiânia, Brazil) Rudini Sampaio (UFC, Fortaleza, Brazil)

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Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Contents

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity

► Given a graph G, a family C of subsets of V(G) is a convexity on G if

- ▶ $\emptyset, V(G) \in C$
- C is closed under intersection
- Every member of C is a convex set
- The convex hull of S is the smallest convex set containing S, denoted by hull(S).

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P₃-hull number P₃-convexity number Other results

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Inapproximability results for graph convexity parameters

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P3-Convexity P₃-hull number P₃-convexity number Other results

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P₃-hull number P₃-convexity number Other results

Convexities (P_3 , geodetic, monophonic, m^3)

- P₃-convexity all paths with 3 vertices
- Geodetic convexity all shortest paths
- Monophonic convexity all induced paths
- m³-convexity all induced paths of size at least 3

Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity

Intervals (P_3 , geodetic, monophonic, m^3)

Given a graph G and a set S of vertices, let:

- $I_{P_3}(S) = S \cup \{P_3$'s between vert. of $S\}$
- ▶ $I_{geo}(S) = S \cup \{\text{minimum paths between vert. of } S\}$
- ▶ $I_{mo}(S) = S \cup \{ \text{induced paths between vert. of } S \}$
- ▶ $I_{m^3}(S) = S \cup \{ \text{induced paths length} \geq 3 \text{ between vert. of } S \}$

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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

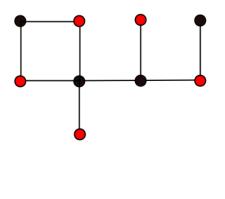
P₃-Convex hull

Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity



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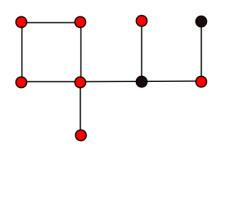
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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity



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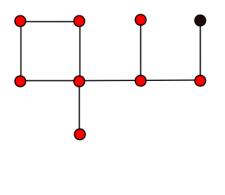
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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity



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Some definitions

► *S* is a hull set

if
$$hull(S) = V(G)$$

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Convexity parameters

Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

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Convexity parameters

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

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Convexity parameters

• hull number $\min_{S} |S|$ s.t. S is a hull set

Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P₃-hull number P₃-convexity number Other results

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P₃-hull number P₃-convexity number Other results

Complexity results

Geodetic convexity (NP-hard results)

- Hull number [Araújo et al., 2013, TCS]
- Interval number [Dourado et al., 2010, DM]
- ► Convexity number [Dourado et al., 2012, G&C]
- Carathéodory number [Dourado et al., 2013, sub]

P₃ convexity (NP-hard results) (bipartite graphs)

- ► Hull number [Centeno et al., 2011, TCS]
- Interval number [Centeno et al., 2009, ENDM]
- Convexity number [Centeno et al., 2009, ENDM]
- Carathéodory number [Barbosa et al., 2012, SIAM J.DM]

Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Our results

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

L-Reduction from MAX-2-SAT-3 (APX-Complete)

- Every clause has at most 2 literals,
- Every literal is in some clause and,
- ▶ For every variable x_i, there are at most 3 clauses containing either x_i or x_i.

Example $\Phi = (x_1 \lor x_2) \land (x_1 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2)$ Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity

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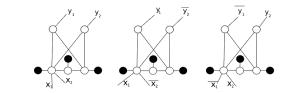
Example

 $\Phi = (x_1 \lor x_2) \land (x_1 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2)$

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results



Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

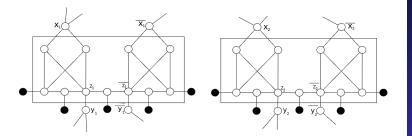
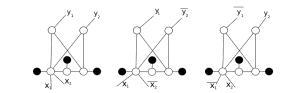


Figura: Max-2-Sat-3 formula: $\Phi = (x_1 \lor x_2) \land (x_1 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2)$

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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

Geodetic convexity

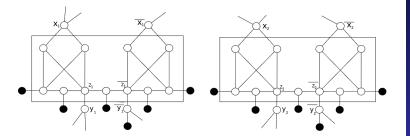


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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

Every hull set contains

- all black vertices = b
- one vertex for every variable gadget

hull number is at least b + k + 3m

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

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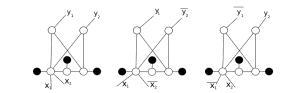
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Inapproximability results for graph convexity parameters

Convexity in graphs

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Geodetic convexity

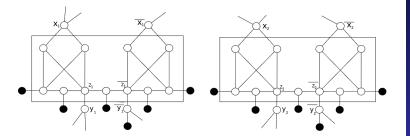
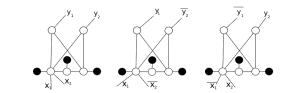


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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

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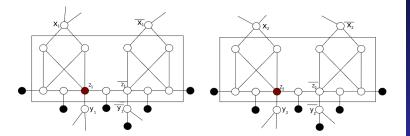
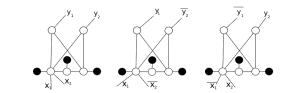


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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

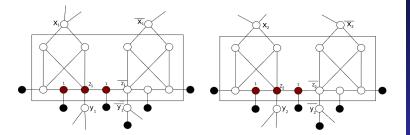
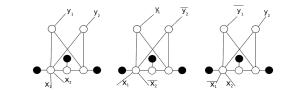


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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

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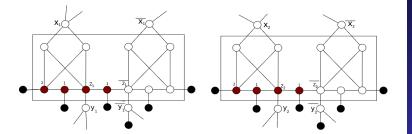
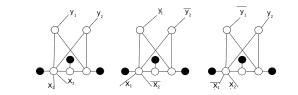


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Geodetic convexity

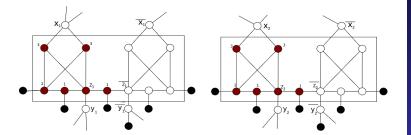
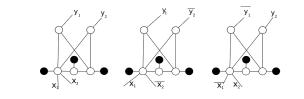


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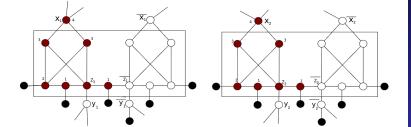


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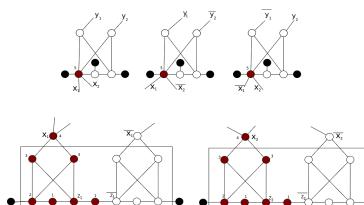
Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

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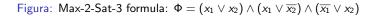


Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity



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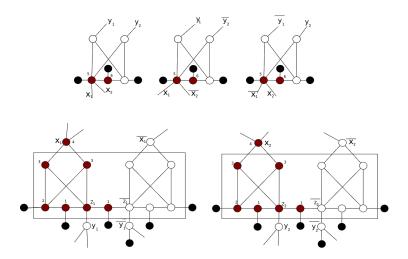


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Inapproximability

results for graph convexity parameters

P2-hull number

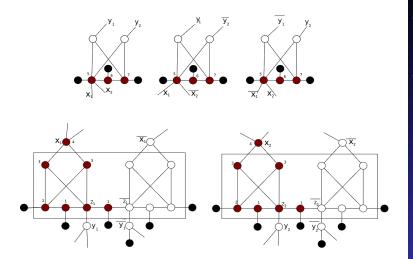


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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

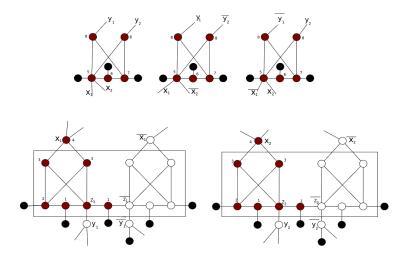


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Inapproximability

results for graph convexity parameters

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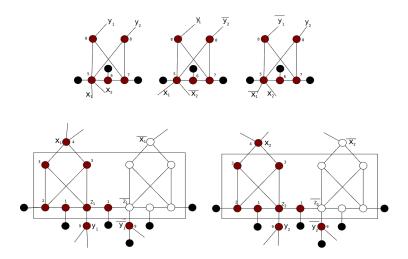


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Inapproximability

results for graph convexity parameters

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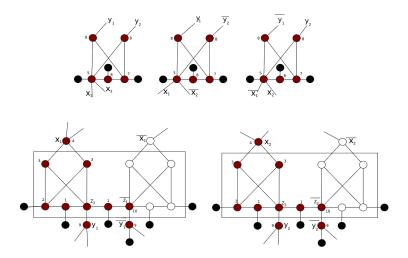


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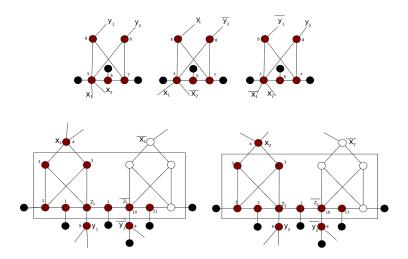


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Inapproximability results for graph convexity parameters

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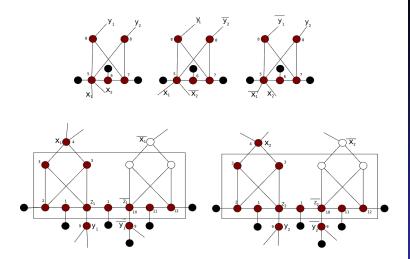


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Inapproximability results for graph convexity parameters

Convexity in graphs

P3-Convexity P3-hull number P3-convexity number Other results

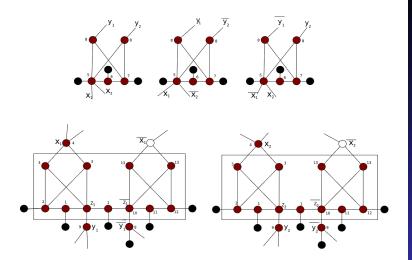


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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

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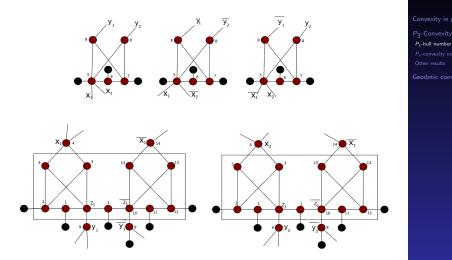


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Inapproximability

results for graph convexity parameters

P₃-hull number is APX-hard in bipartite graphs

• hull number is at least b + k + 3m

Every hull set of size b + k + 3m + ℓ defines an assignment satisfying m − ℓ clauses and vice versa Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

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P₃-hull number is APX-hard in bipartite graphs

- hull number is at least b + k + 3m
- ► Every hull set of size $b + k + 3m + \ell$ defines an assignment satisfying $m \ell$ clauses and vice versa

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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

 $O(n^{1-\varepsilon})$ -innaproximable in bipartite graphs in polynomial time unless P=NP.

Reduction from SET-PACKING $(O(n^{1-\varepsilon})$ -innaproximable problem)

Given *m* sets S_1, \ldots, S_m , determine the maximum *k* s.t. there exist *k* pairwise disjoint sets.

Example (k = 3, m = 5)

▶
$$S_1 = \{a, b, c\}, \quad S_2 = \{b, f, g\}$$

•
$$S_3 = \{a, e, i\}, \quad S_4 = \{c, e, g\}$$

• $S_5 = \{g, h, i\}$

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•
$$S_3 = \{a, e, r\}, \quad S_4 = \{c, e, g\}$$

• $S_5 = \{g, h, i\}$

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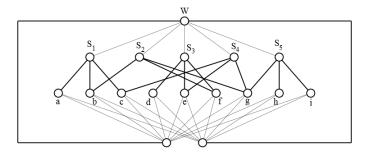


Figura: Reduction from SET-PACKING

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Inapproximability results for graph convexity parameters

Convexity in graphs

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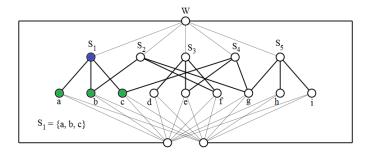


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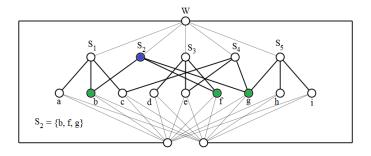


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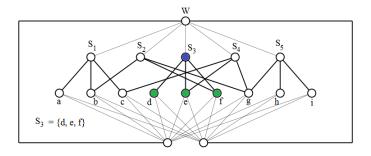


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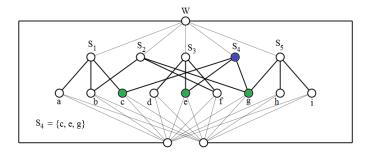


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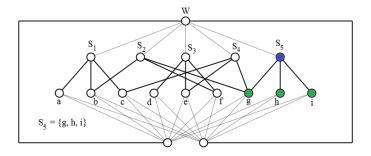


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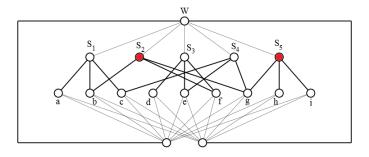


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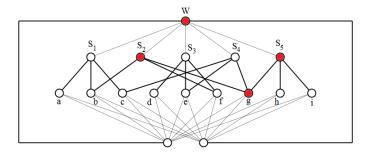


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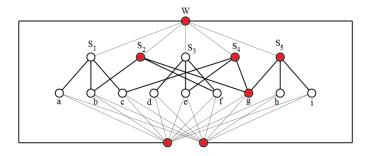


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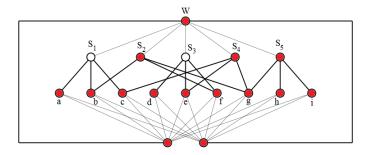


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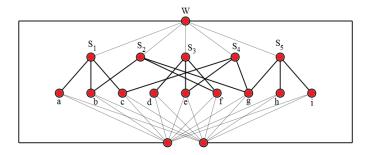


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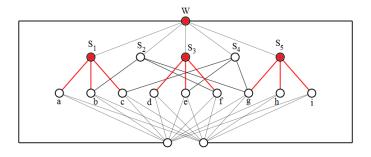


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Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

- A partition S₁ ∪ S₂ of a set S is a Radon partition of S if hull(S₁) ∩ hull(S₂) ≠ Ø
- The Radon number of a graph G is the smallest k for which every S ⊂ V(G), with size at least k, admits a Radon partition

Inapproximability results for graph convexity parameters

Convexity in graphs

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Inapproximability results for graph convexity parameters

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Geodetic convexity

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Inapproximability results for graph convexity parameters

Convexity in graphs

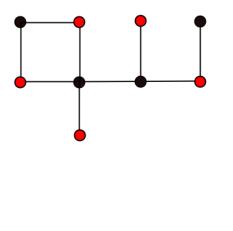
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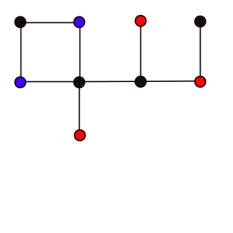
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Inapproximability results for graph convexity parameters

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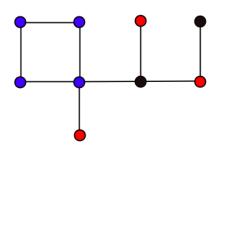
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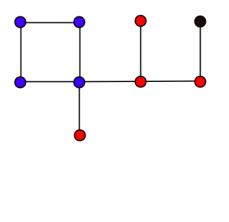
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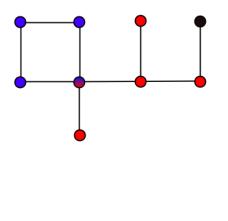
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Geodetic convexity



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- The P₃-Radon number problem is O(n^{1-ε})-innaproximable in bipartite graphs in polynomial time unless P=NP.
- From SET-PACKING problem

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

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Innaproximability of P3-interval number

The P₃-interval number problem is O(log n)-innaproximable in bipartite graphs in polynomial time unless P=NP.

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From SET COVER problem

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Innaproximability of P3-Carathéodory number

- P₃-Carathéodory number probem is O(n^{1-ε})-innaproximable in bipartite graphs in polynomial time unless P=NP.
- From MAX3SAT-INTERVAL

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

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Geodetic convexity

Inapproximability results for graph convexity parameters

Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

Geodetic convexity

 The results for hull number, Carathéodory number, interval number and convexity number can be extended to geodetic convexity

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Geodetic convexity

Theorem

Let G_1 be a triangle free graph with at least three vertices. Then

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(i)
$$hn_{gd}(G_1 + K_m) = hn_{P_3}(G_1)$$
,
(ii) $in_{gd}(G_1 + K_m) = in_{P_3}(G_1)$,
(iii) $cx_{gd}(G_1 + K_m) = cx_{P_3}(G_1) + m$,
(iv) $cth_{gd}(G_1 + K_m) = \max\{cth_{P_3}(G_1), 2\}$.

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Convexity in graphs

P₃-Convexity P₃-hull number P₃-convexity number Other results

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Inapproximability results for graph convexity parameters

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Geodetic convexity

Geodetic Radon number probem is O(n^{1-ε})-innaproximable in general graphs in polynomial time unless P=NP.

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From MAXIMUM-CLIQUE problem

Thank you

Inapproximability results for graph convexity parameters

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► Thank you!