Identifying codes in grids

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PARGO, May 13, 16h-17h

Identifying code

- Grafo G e um conjunto $C \subseteq V(G)$
- Vizinhança fechada de um vértice v: $N[v] = N(v) \cup \{v\}$.
- C-código de v: $C[v] = N[v] \cap C$
- C é um código de identificação se todos os vértices tem C-códigos não-vazios distintos.

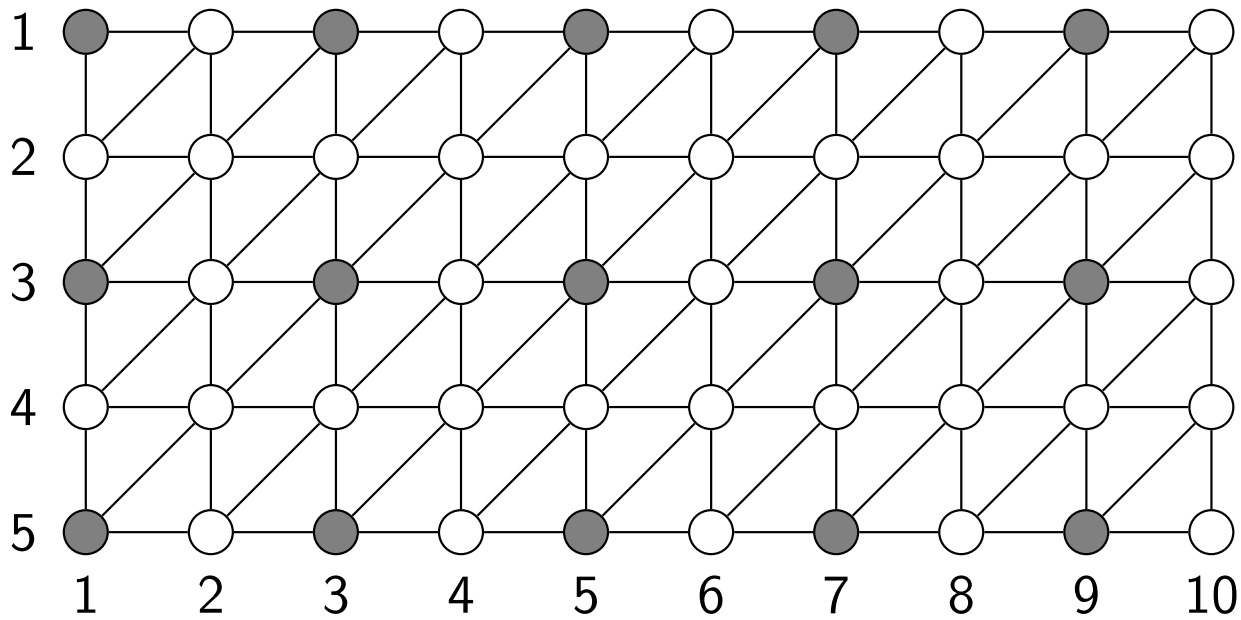

Figura: Código do grid triangular com densidade 1/4

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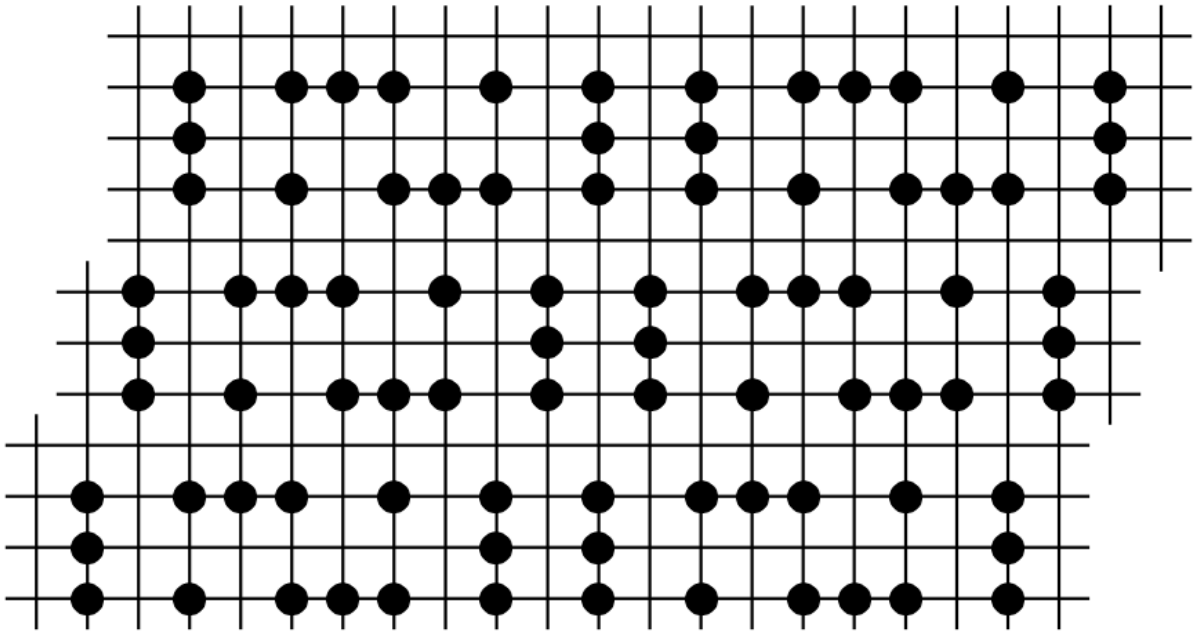

Figura: Código do grid retangular com densidade $7/20$

Identifying code

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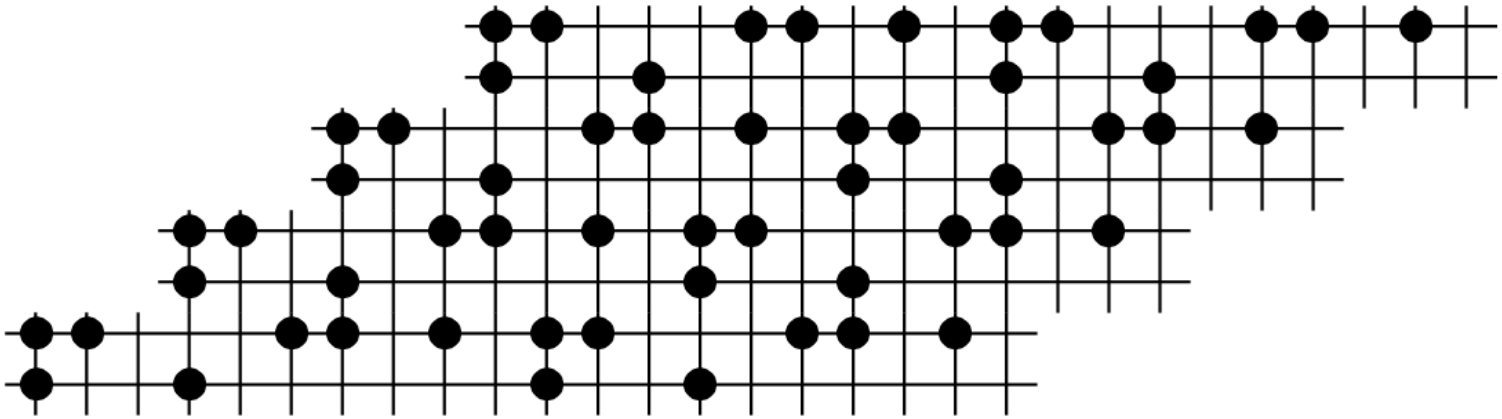

Figura: Outro código do grid retangular com densidade $7/20$

Identifying code

- $C[v] \neq \emptyset$ e $C[v] \neq C[u]$ para todo $u \neq v$.

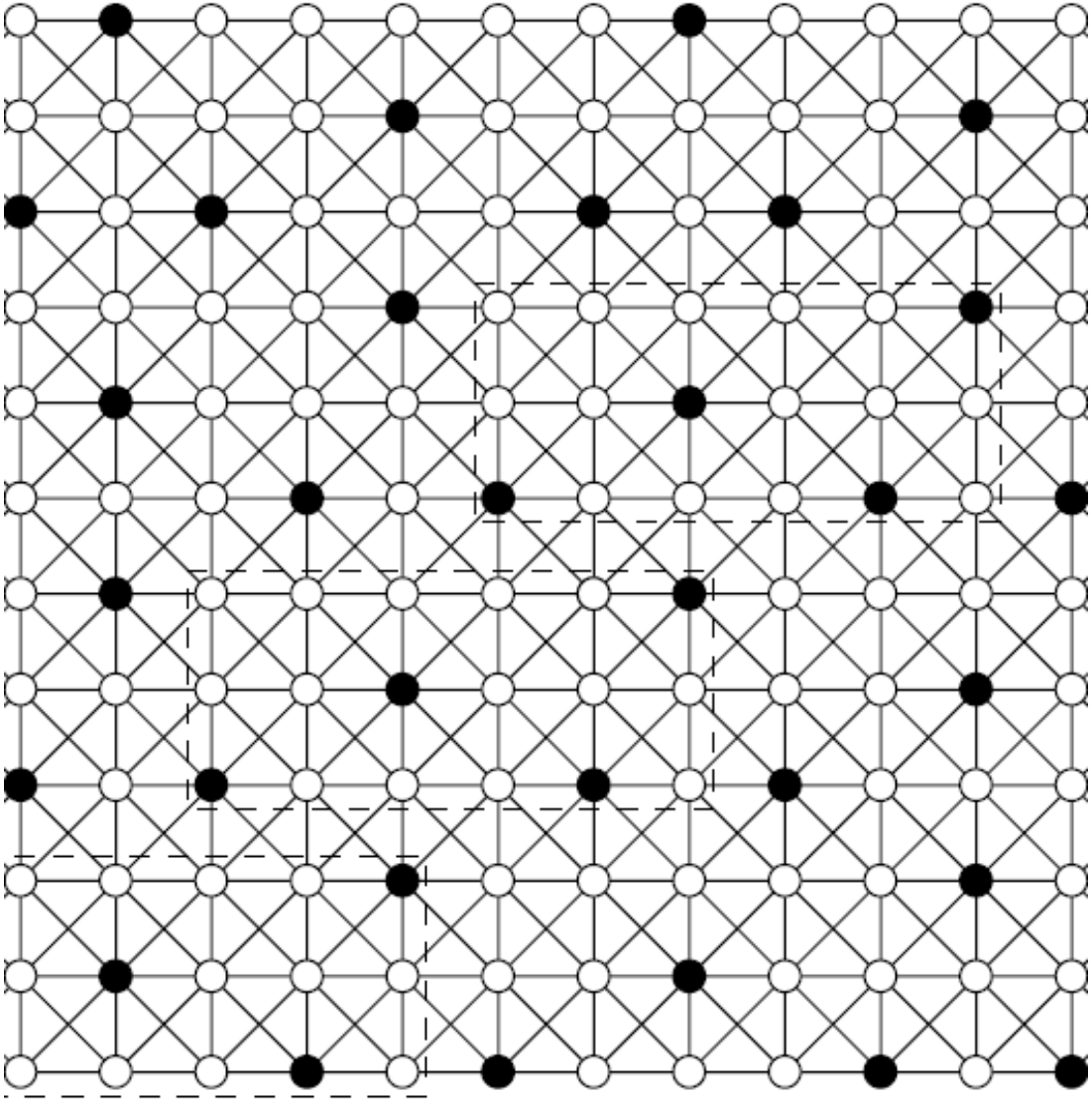

Figura: Código do king grid com densidade 2/9

Identifying code

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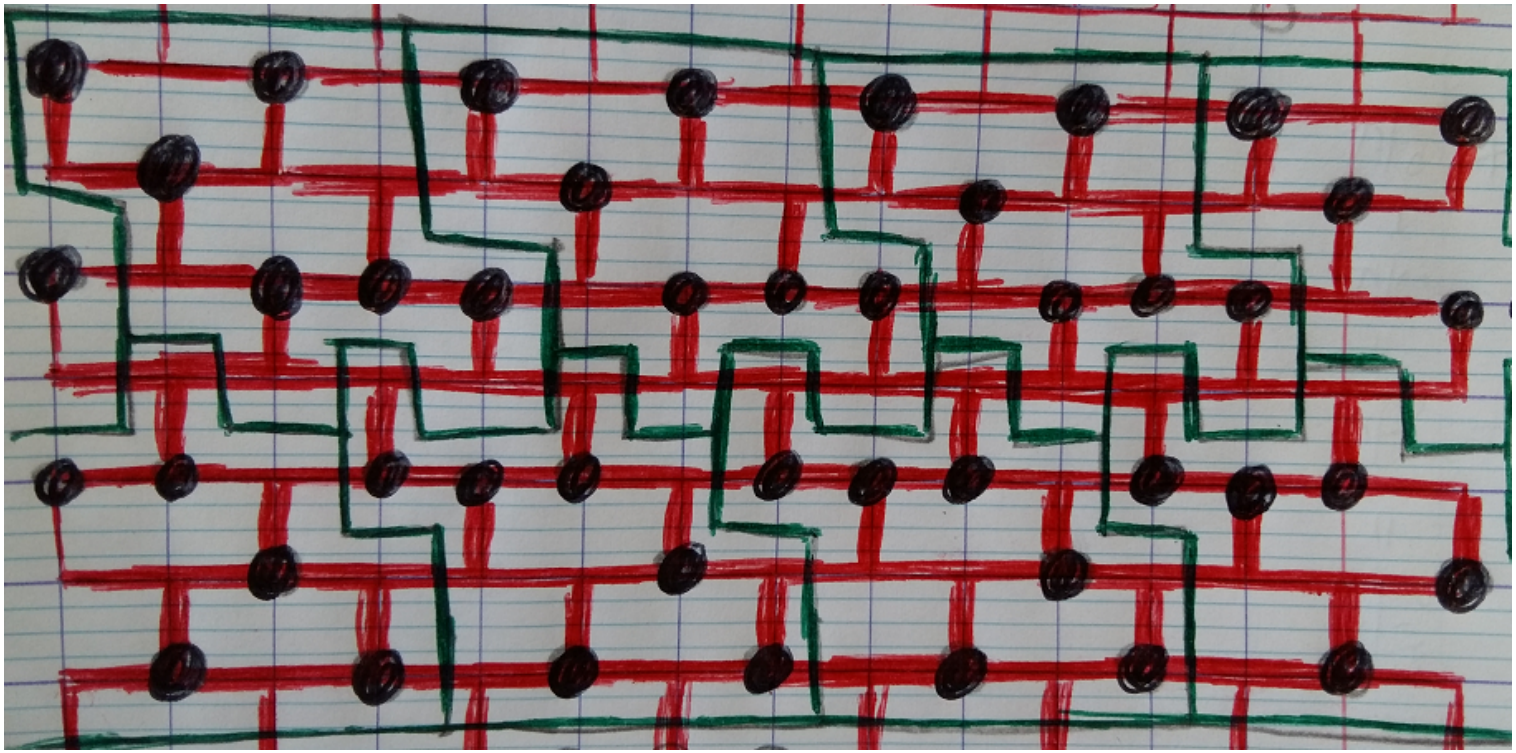

Figura: Código do grid hexagonal com densidade 3/7

Known Results

Grid triangular $\mathcal{G}_T$:

- [Karpovsky et al.,1998] $d^*(\mathcal{G}_T) = 1/4$.

Grid retangular $\mathcal{G}_S$:

- [Cohen et al., 1999] $d^*(\mathcal{G}_S) \leq 7/20$.
- [Ben-Haim, Litsyn, 2005] $d^*(\mathcal{G}_S) = 7/20$.
- [Daniel, Gravier, Moncel, 2004] $d^*(S_1) = 1/2$, $d^*(S_2) = 3/7$, $\frac{7}{20} - \frac{1}{2k} \leq d^*(S_k) \leq \min\left\{\frac{2}{5}, \frac{7}{20} + \frac{2}{k}\right\}$
- [Bouznif et al., 2014] $d^*(S_3) = 3/7$, $\frac{7}{20} + \frac{1}{20k} \leq d^*(S_k) \leq \min\left\{\frac{2}{5}, \frac{7}{20} + \frac{3}{10k}\right\}$.

Grid hexagonal $\mathcal{G}_H$:

- [Cohen et al., 2000] $16/39 \leq d^*(\mathcal{G}_H) \leq 3/7$.
- [Cranston, Yu, 2009] $12/29 \leq d^*(\mathcal{G}_H) \leq 3/7$.
- [Cukierman, Yu, 2013] $5/12 \leq d^*(\mathcal{G}_H) \leq 3/7$.

King Grid $\mathcal{G}_K$:

- [Charon et al., 2002] $d^*(\mathcal{G}_K) = 2/9$.

New results on Triangular Grids

[Rennan, Havet, Rudini, 2015]

- Triangular grid T_k with k rows:
- $d^*(T_2) = 1/2$
- $d^*(T_3) = 1/3$
- $d^*(T_4) = 1/3$
- $d^*(T_5) = 3/10$
- $d^*(T_6) = 1/3$
- $d^*(T_k) = 1/4 + 1/(4k)$ for all $k \geq 7$ odd
- $1/4 + 1/(4k) \leq d^*(T_k) \leq 1/4 + 1/(2k)$ for all $k \geq 8$ even.

Triangular grid with 2 rows

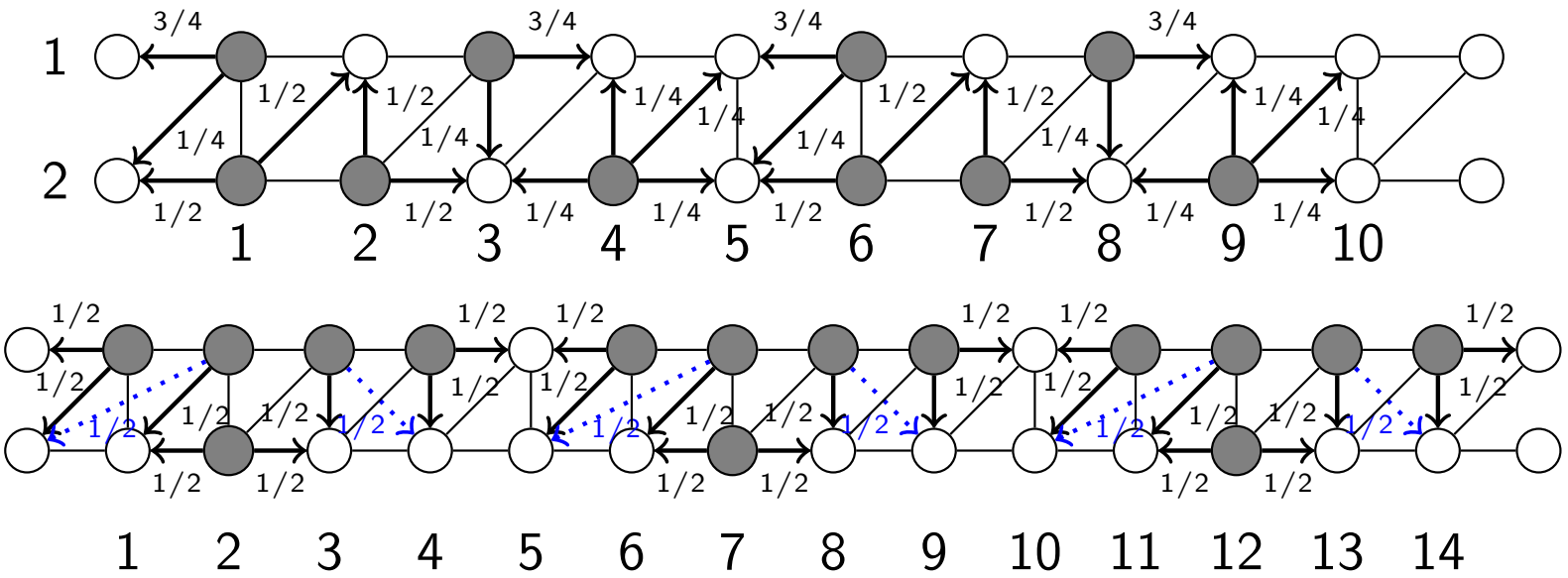

Figura: Identifying codes of T_2 with density 1/2

Triangular grid with 3 rows

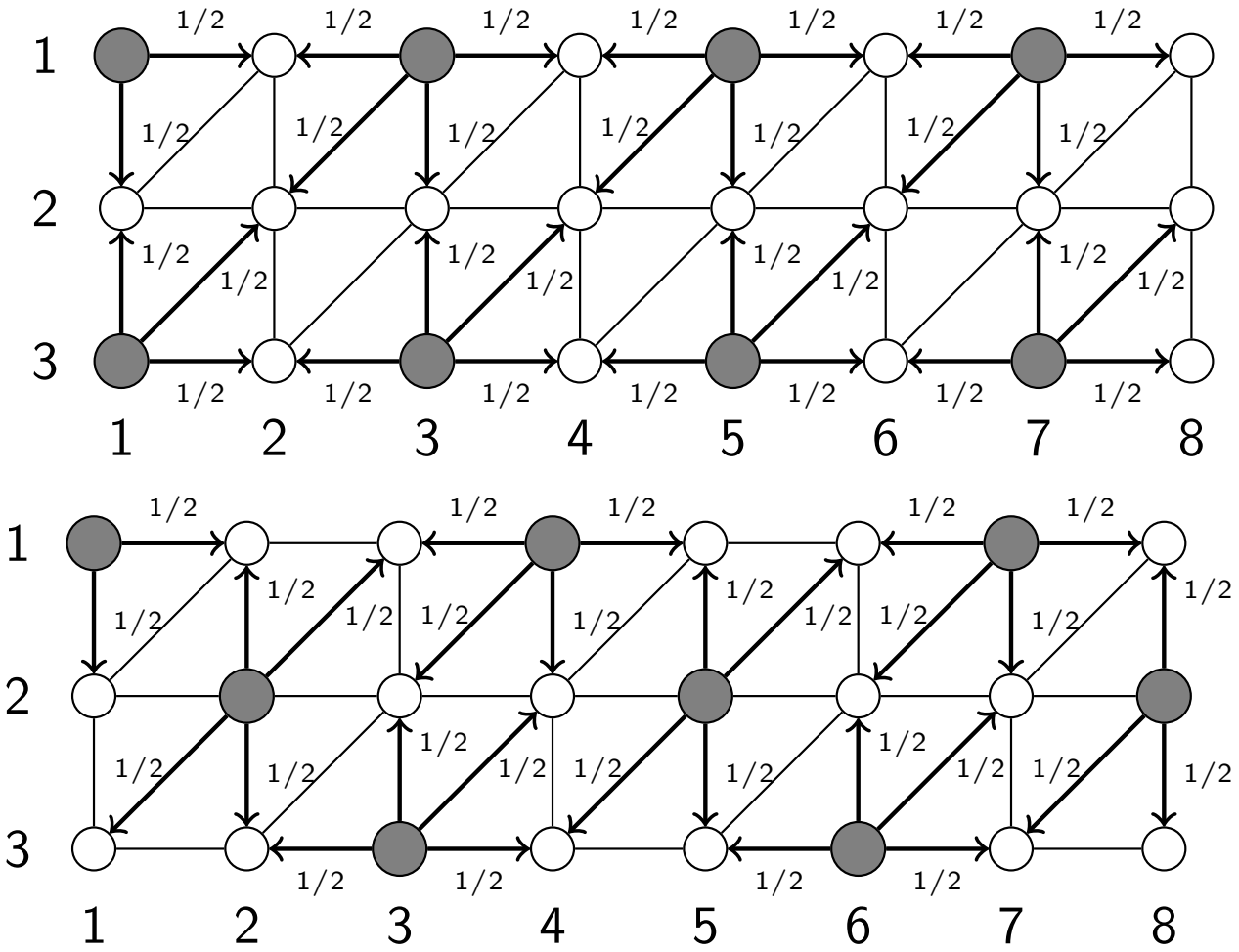

Figura: Identifying codes of T_3 with density $1/3$

Triangular grid with 4 rows

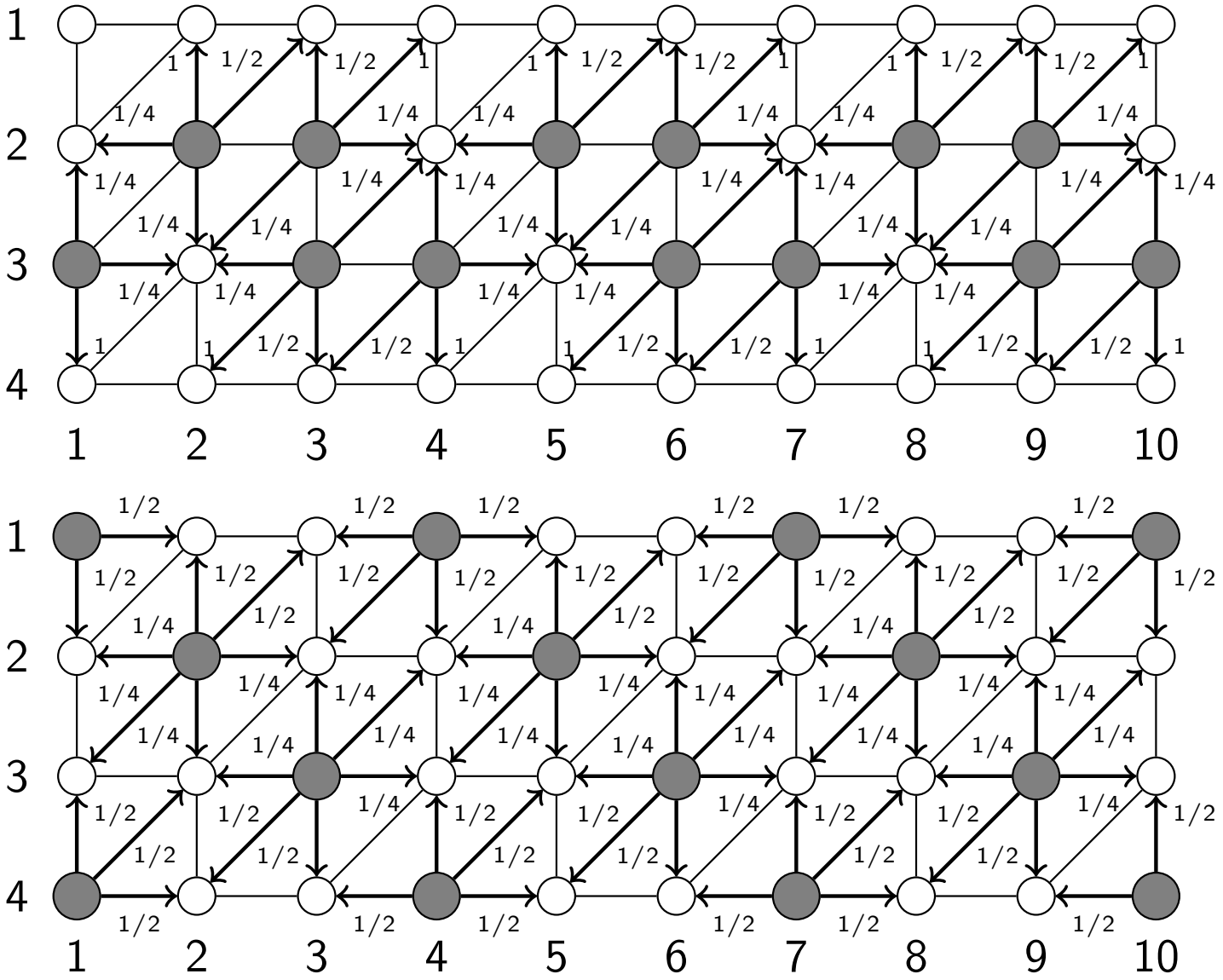

Figura: Two identifying codes of T_4 with density $1/3$

Triangular grid with 5 rows or more (odd)

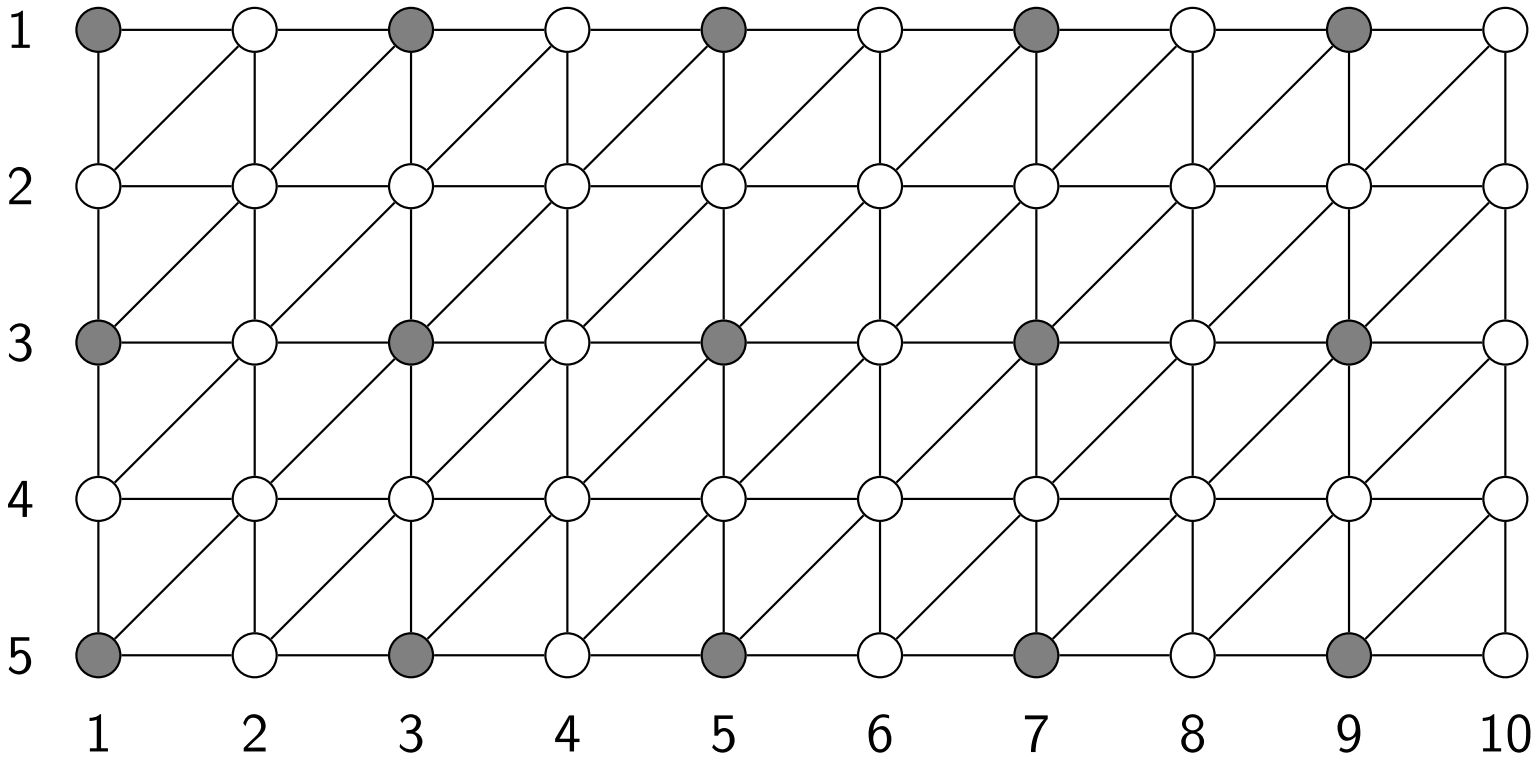

Figura: Identifying code C_5 of T_5 with density 3/10.

Triangular grid with 6 rows or more (even)

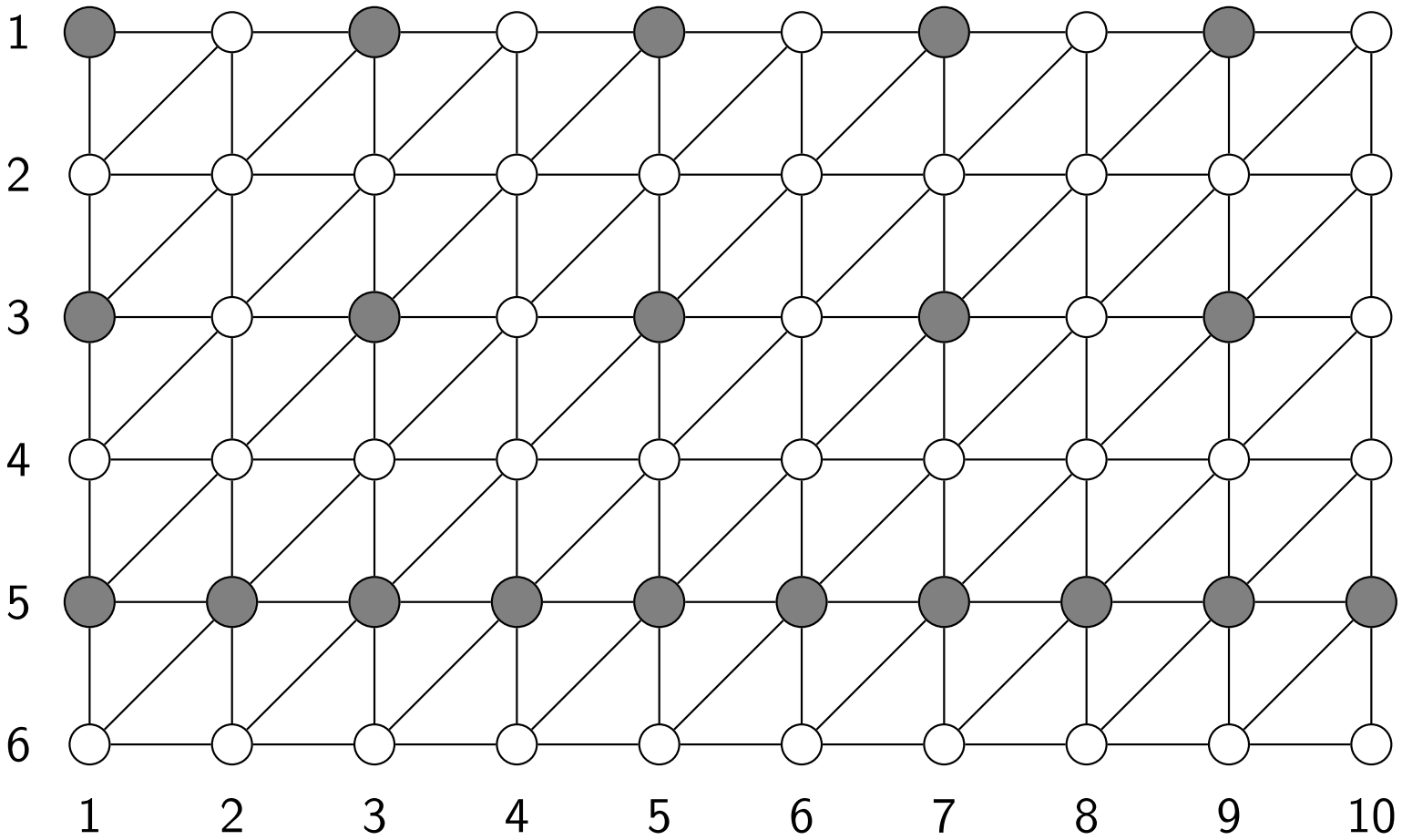

Figura: Identifying code $\mathcal{C}_6$ of T_6 with density $1/3$

New results on King grids

[Juliette, Rennan, Havet, Rudini, 2016]:

- King Grid R_k with k rows
- $d^*(R_3) = 1/3 = 0,3333...$
- $d^*(R_4) = 5/16 = 0,3125$
- $d^*(R_5) = 4/15 = 0,2666...$
- $d^*(R_6) = 5/18 = 0,2777...$
- $d^*(R_n) \geq 2/9 + 5/81n$, for every $n \geq 7$
- $d^*(R_n) \leq 2/9 + 6/18n$, for every $n = 3k \geq 9$
- $d^*(R_n) \leq 2/9 + 8/18n$, for every $n = 3k + 1 \geq 7$
- $d^*(R_n) \leq 2/9 + 7/18n$, for every $n = 3k + 2 \geq 8$

King grid with 3 rows

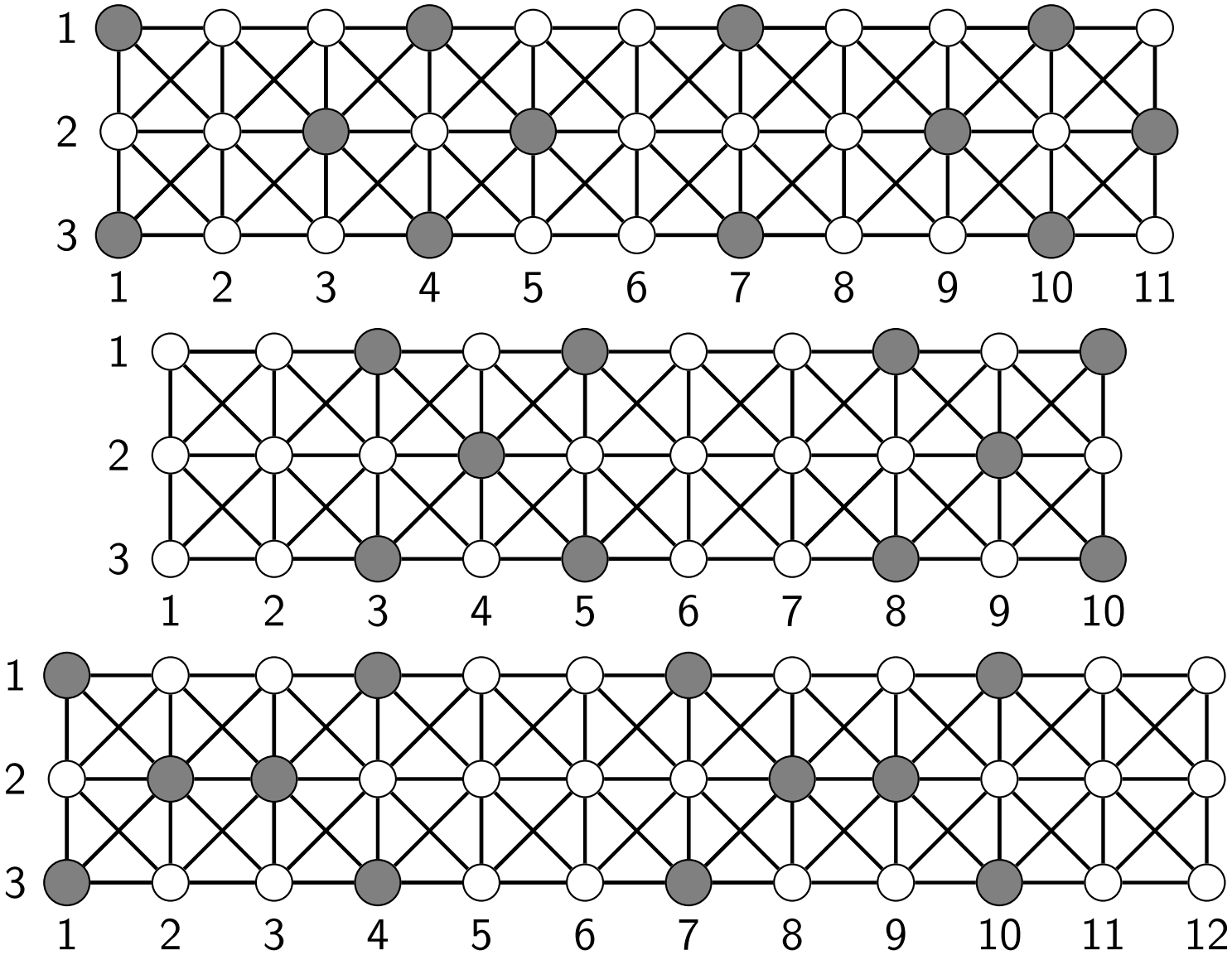

Figura: Id code of King grid with 3 rows and density 1/3

King grid with 3 rows

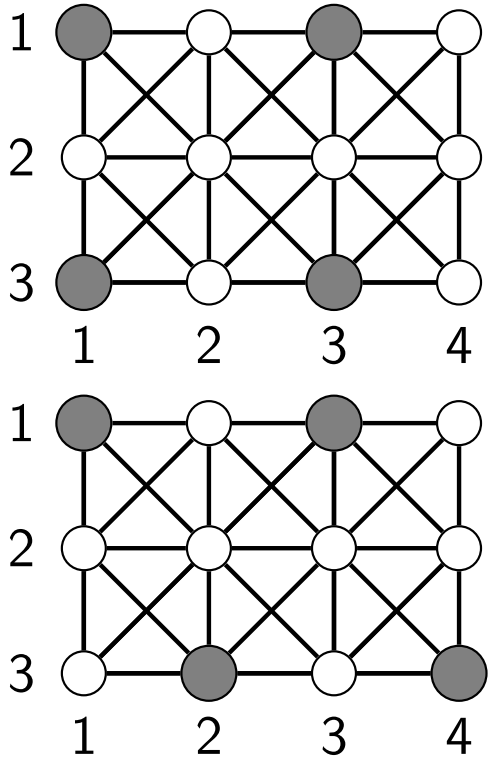

Figura: Id code of King grid with 3 rows and density 1/3

Proof of King grid with 3 rows

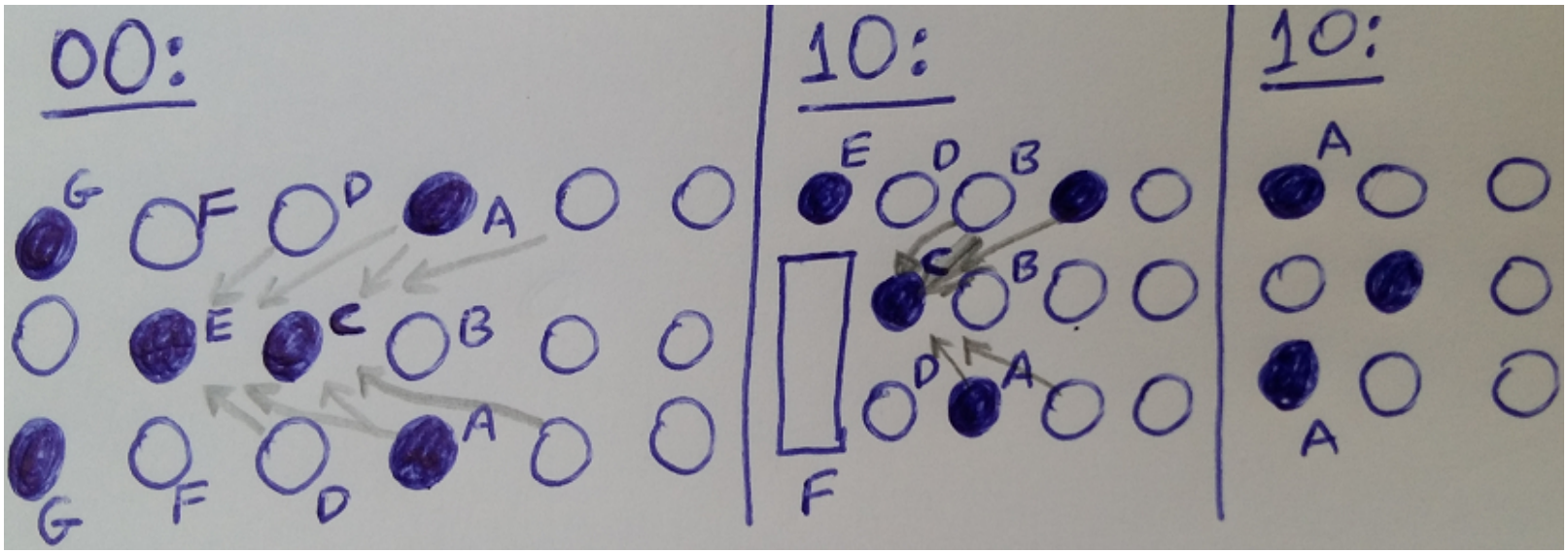

Figura: Proof for king grid with 3 rows

King grid with 4 rows

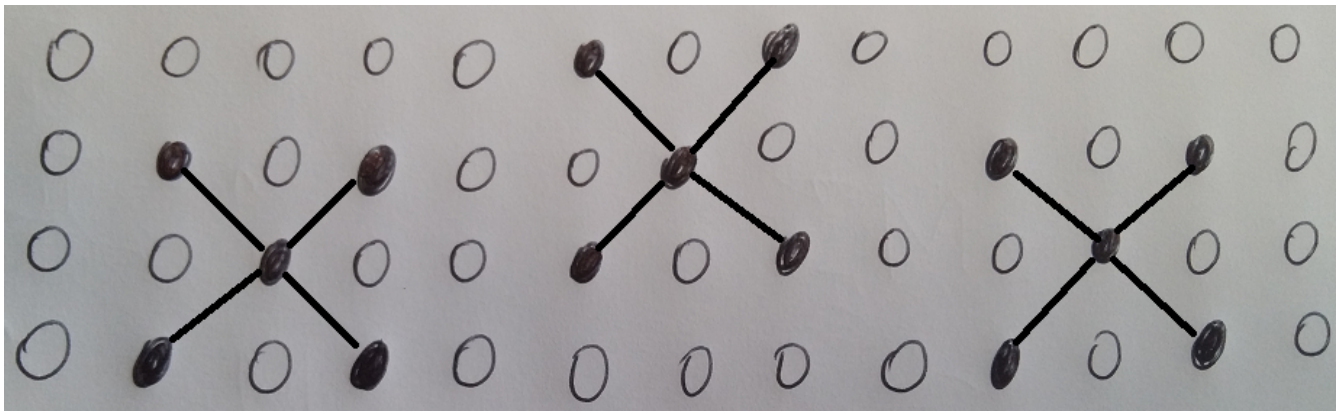

Figura: Id code of King grid with 4 rows and density 5/16

Proof of King grid with 4 rows

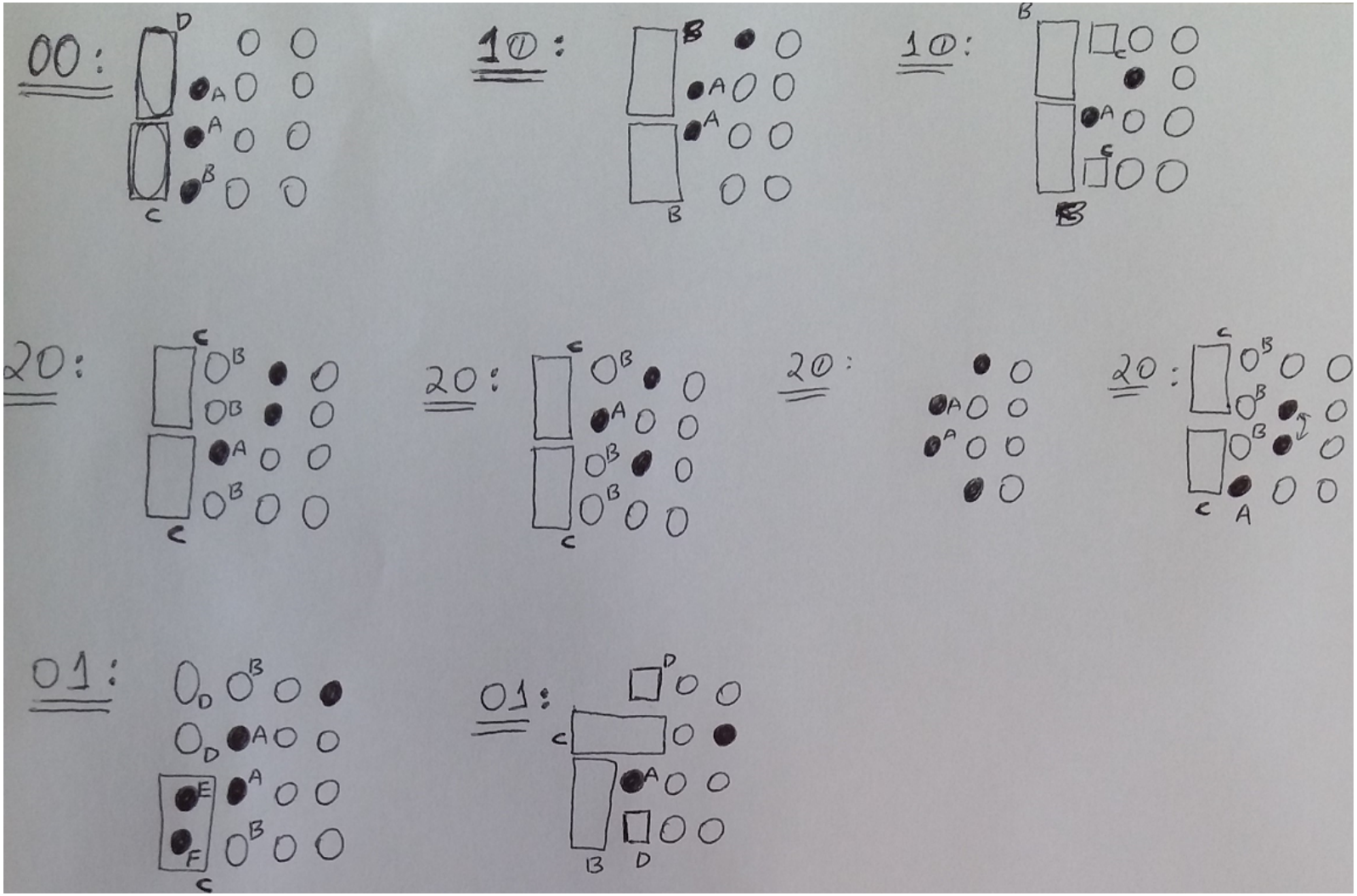

Figura: Proof for king grid with 4 rows - case 1

Proof of King grid with 4 rows

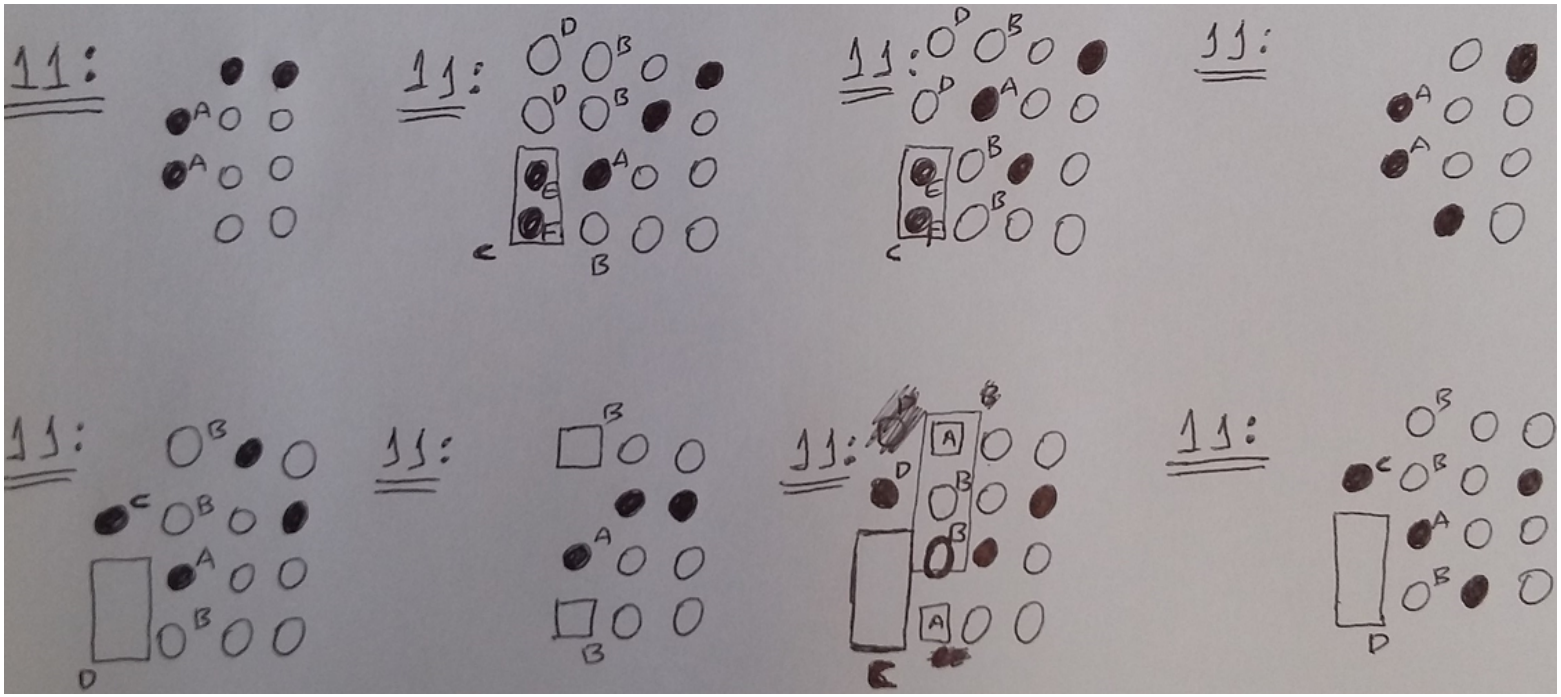

Figura: Proof for king grid with 4 rows - case 2

King grid with 5-6 rows

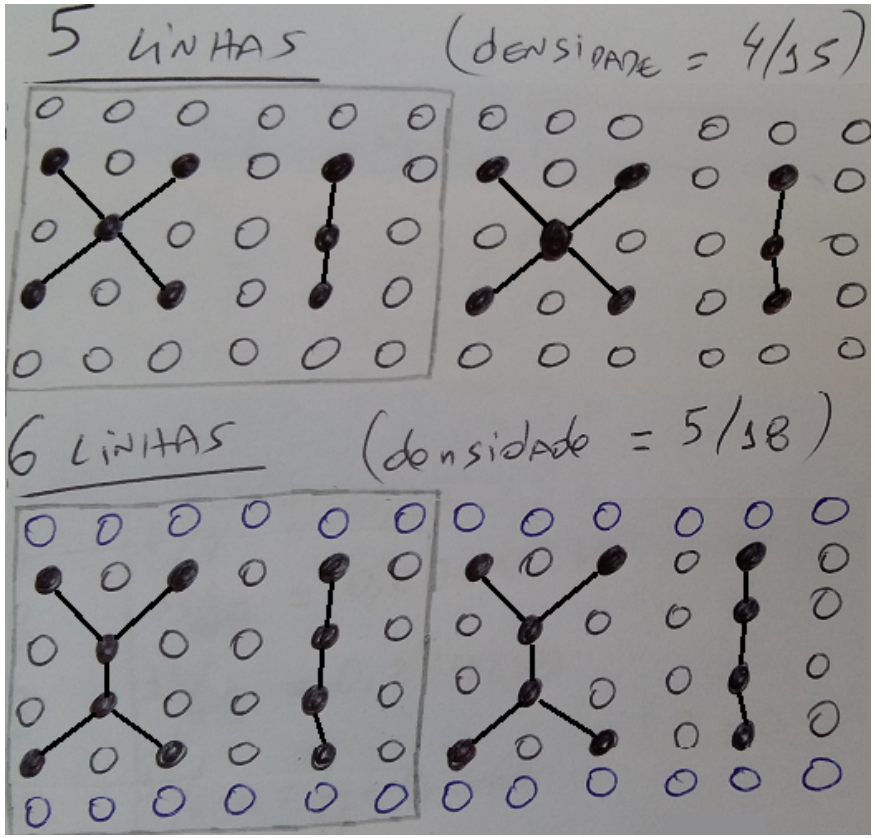

Figura: Optimum Id codes of King grid with 5-6 rows

King grid with 5 rows

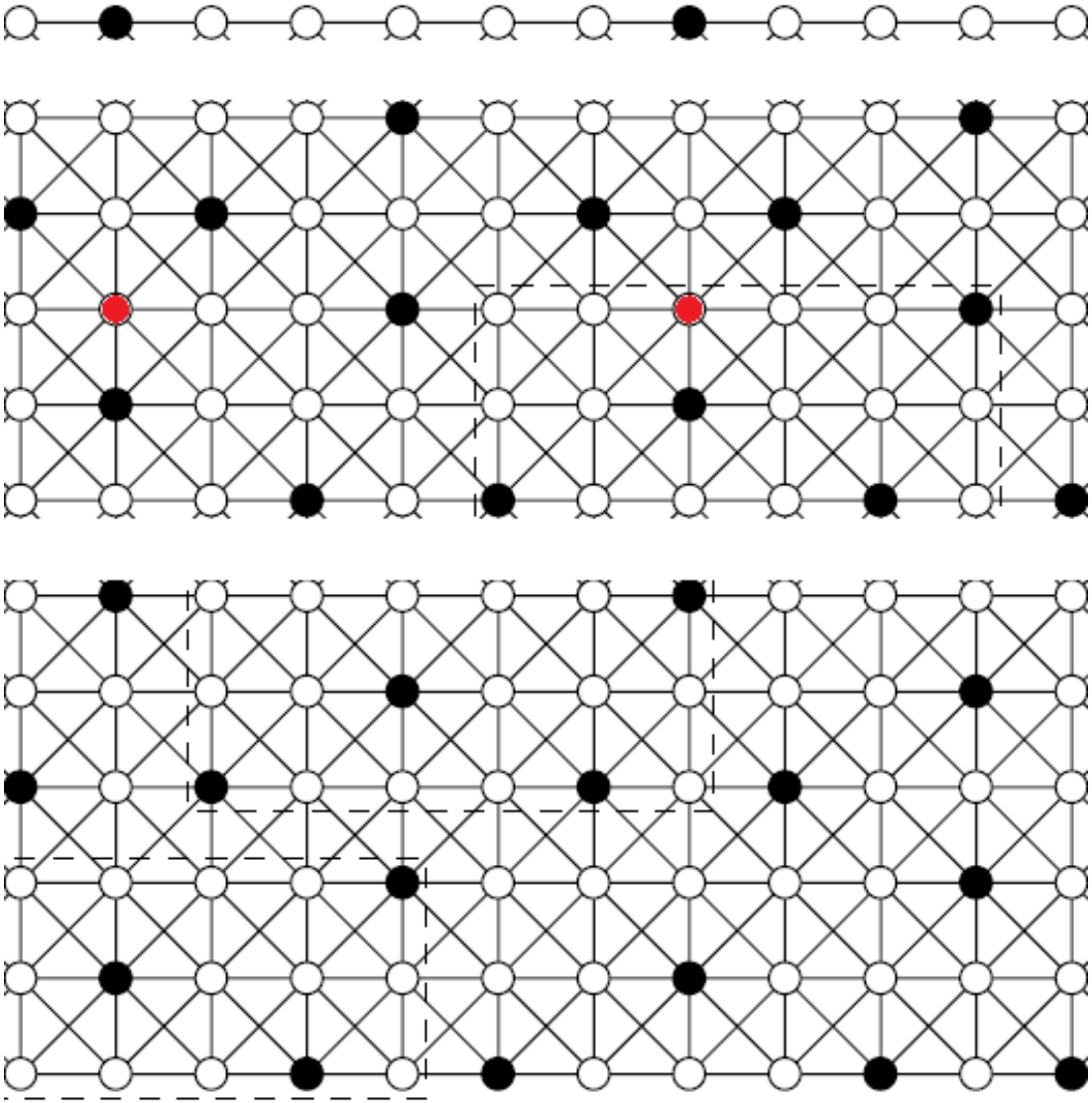

Figura: Optimum Id code of King grid with 5 rows and density 4/15

King grid with 6 rows

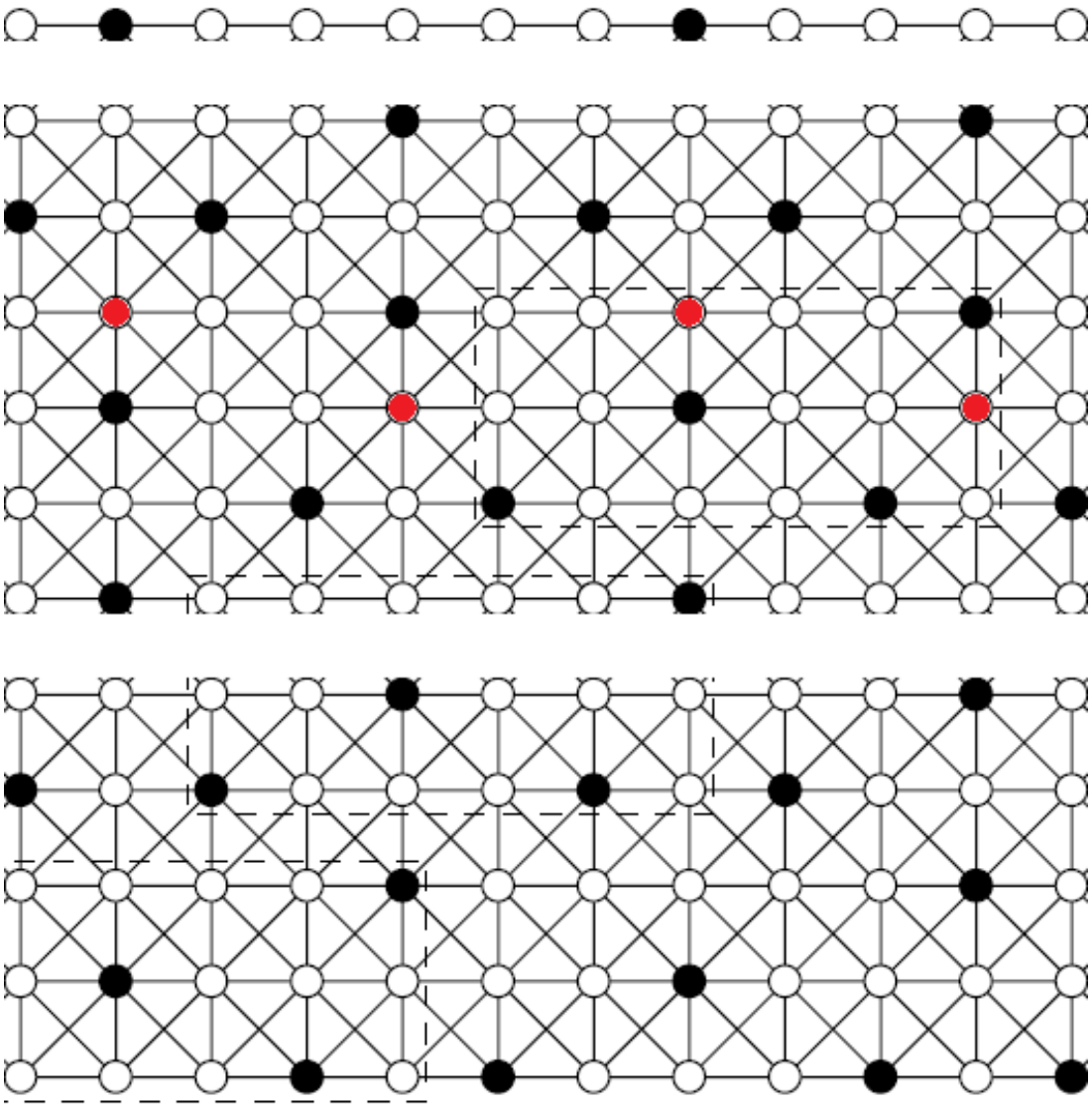

Figura: Optimum Id code of King grid with 6 rows and density 5/18

King grid with 7 rows

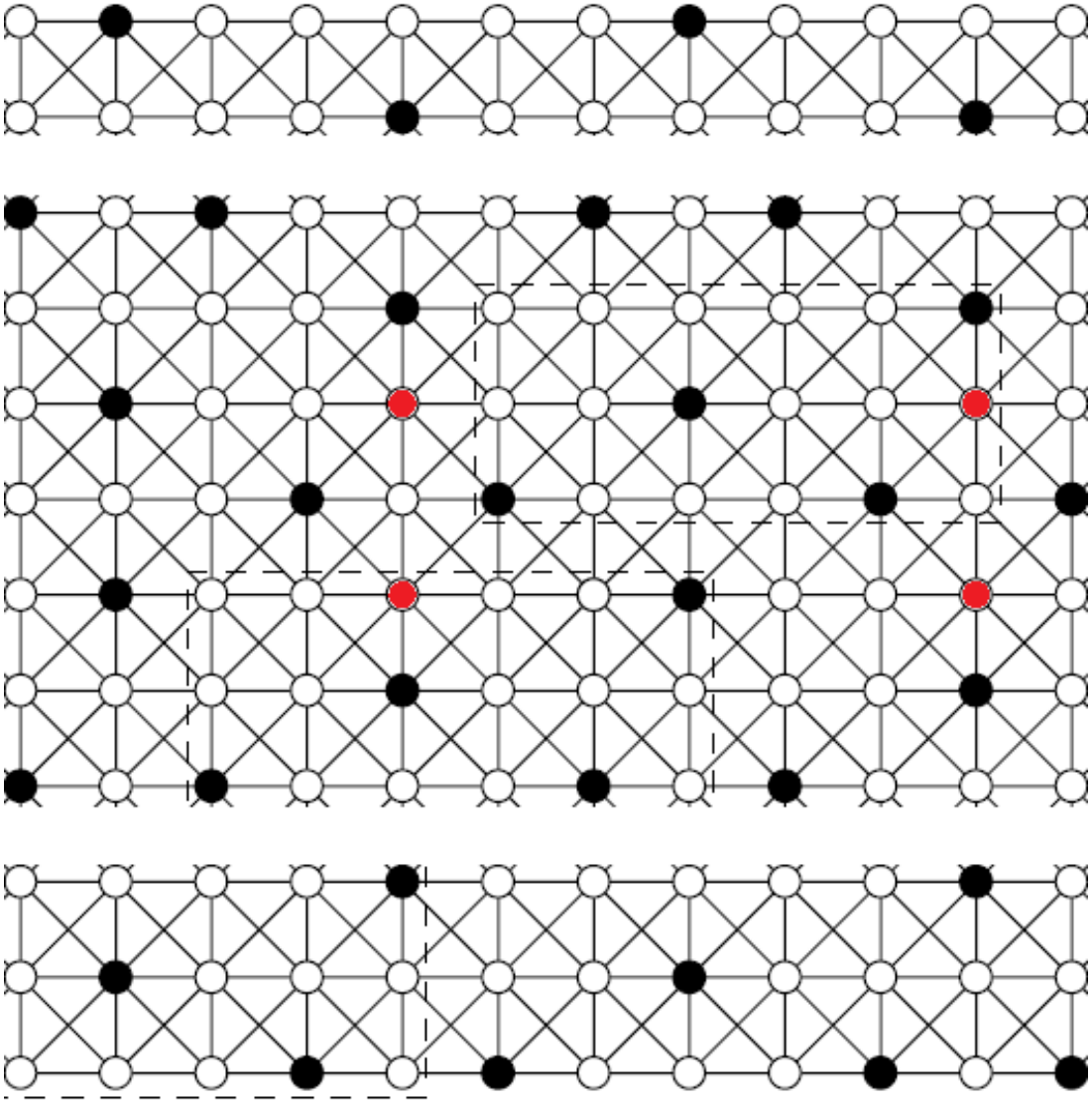

Figura: Id code of King grid with 7 rows

King grid with 8 rows

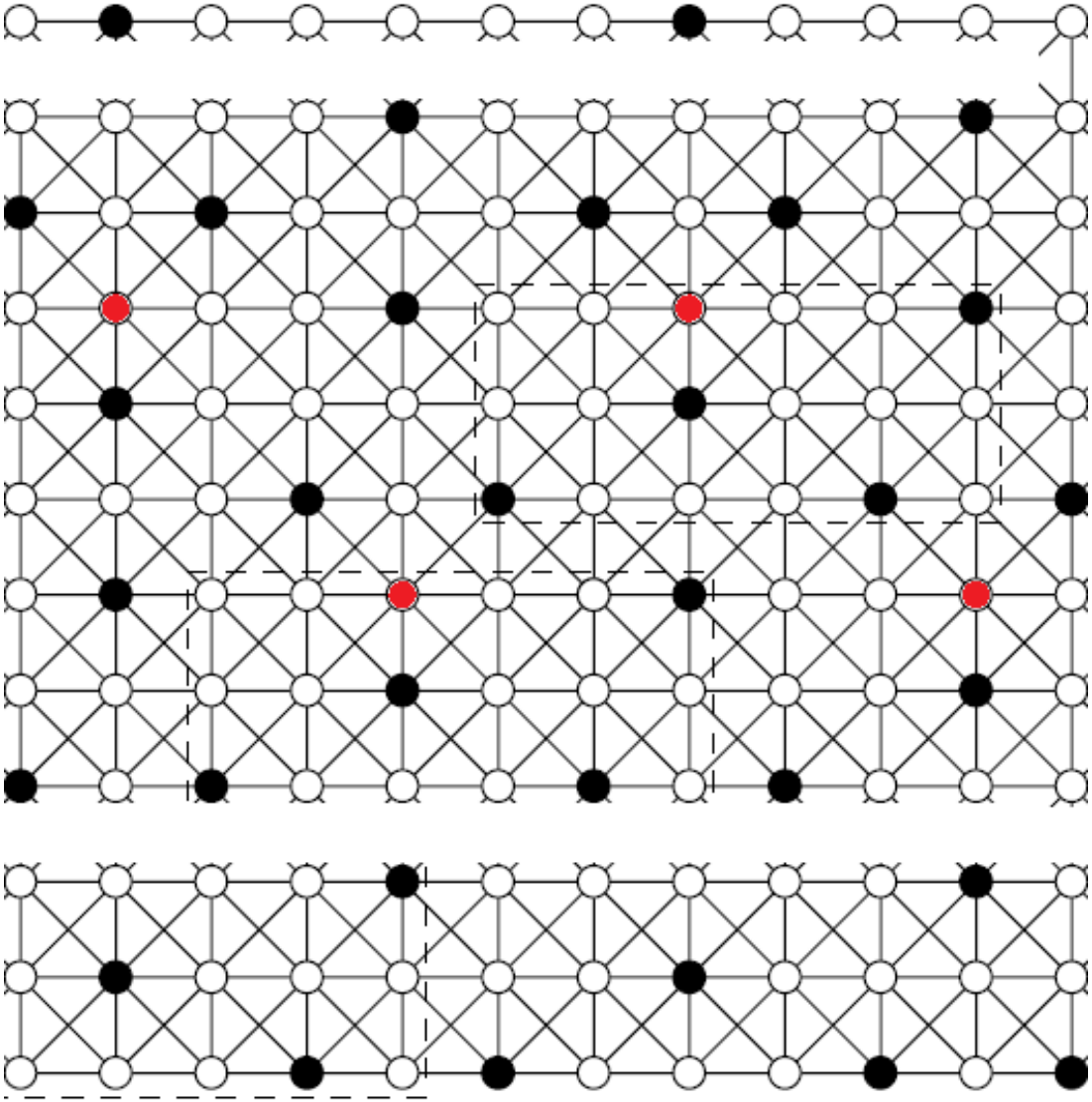

Figura: Id code of King grid with 8 rows

King grid with 9 rows

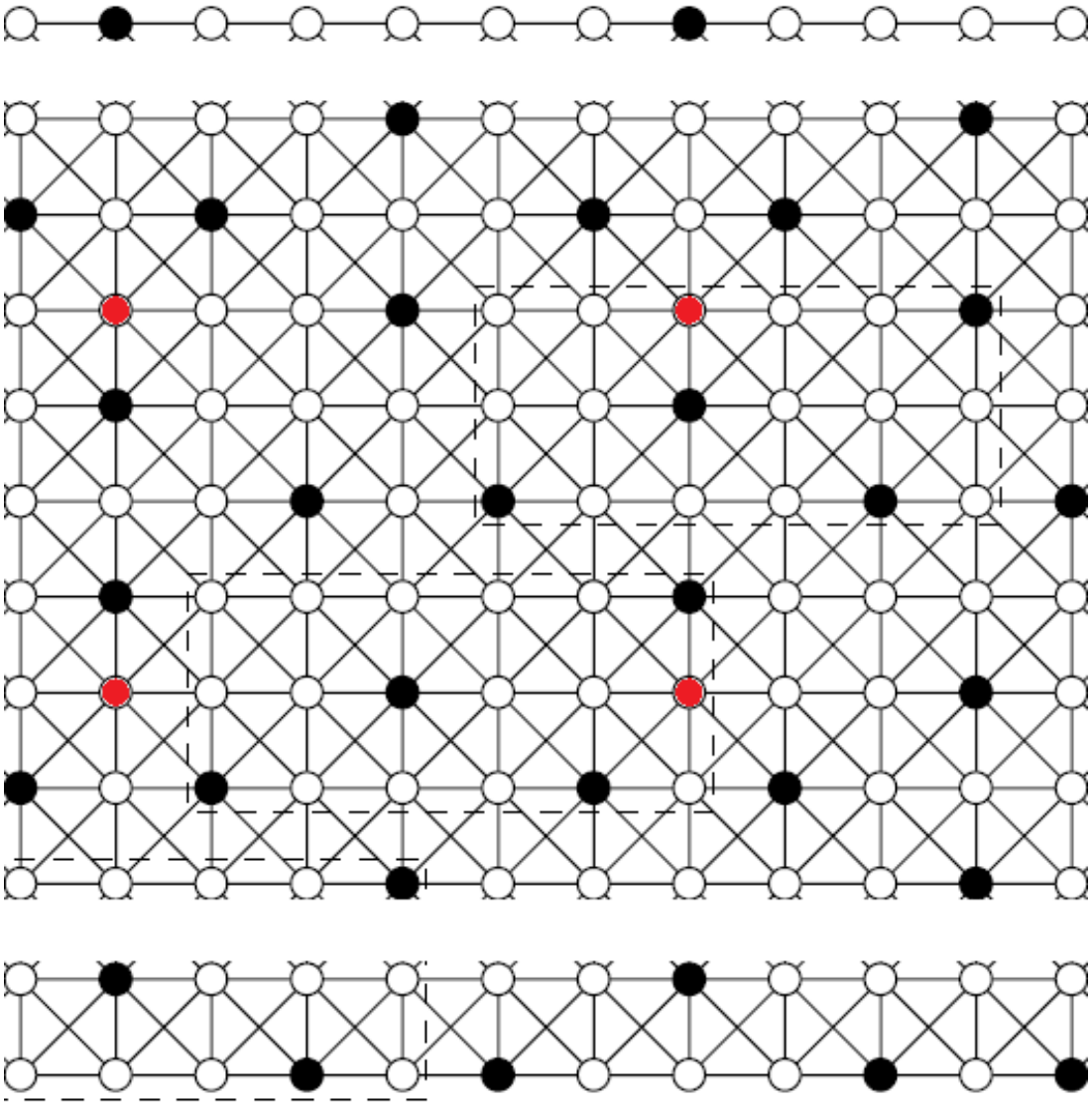

Figura: Id code of King grid with 9 rows

King grid with 10 rows

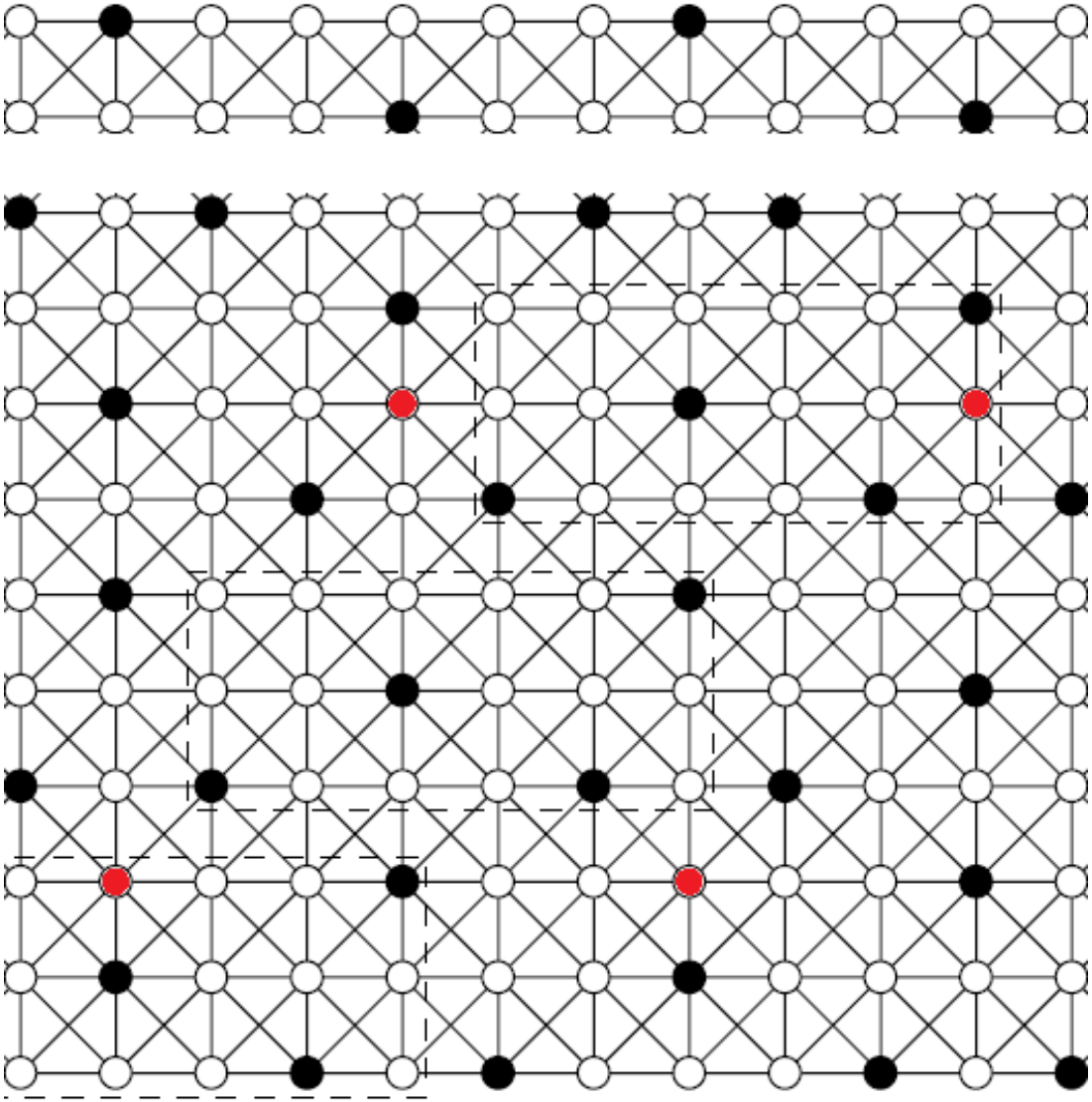

Figura: Id code of King grid with 10 rows

King grid with 11 rows

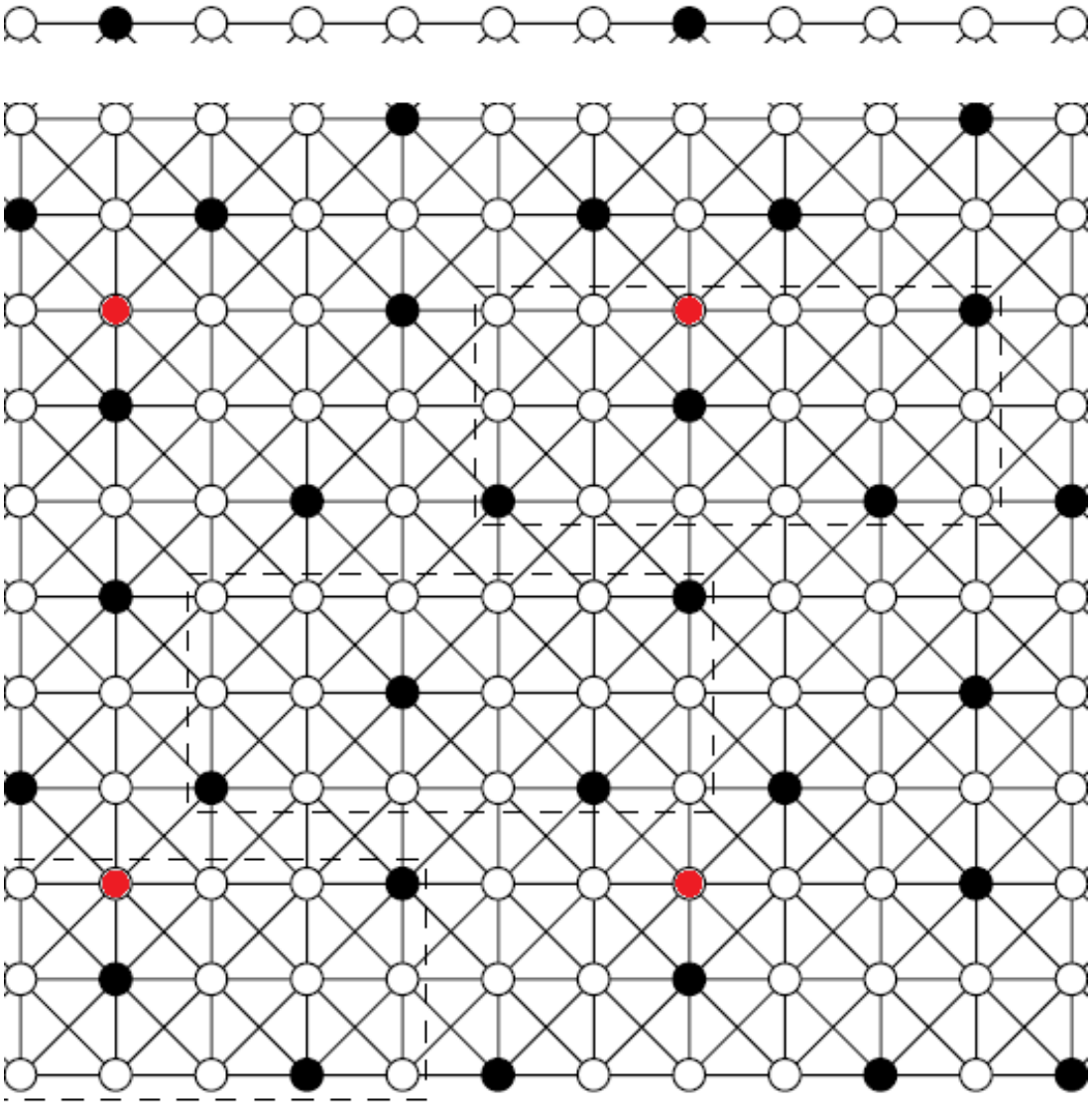

Figura: Id code of King grid with 11 rows