Convexity of induced paths of order 3

Convexity in graphs P3-hull number P3-convexity number

Convexity of induced paths of order 3

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This is a joint work with Rafael Teixeira (UFC, Fortaleza, Brazil) Jayme Szwarcfiter (UFRJ, Rio de Janeiro, Brazil)

LAGOS-2013, Mexico, April 25, 12h00-12h25

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Convexity of induced paths of order 3

Convexity in graphs ²3-hull number ²3-convexity number

Convexity in graphs

P₃-hull number

P₃-convexity number

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Intervals (P_3 , geodesic, monophonic, m^3) Given a graph G and a set S of vertices, let:

- $I_{P_3}(S) = S \cup \{P_3$'s between vert. of $S\}$
- ▶ $I_{geo}(S) = S \cup \{\text{minimum paths between vert. of } S\}$
- ▶ $I_{mo}(S) = S \cup \{ \text{induced paths between vert. of } S \}$
- ▶ $I_{m^3}(S) = S \cup \{ \text{induced paths length} \ge 3 \text{ between vert. of } S \}$

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Convexities (P_3 , geodesic, monophonic, m^3)

- $I_{P_3}(S) = S \iff S$ is P_3 -convex
- $I_{geo}(S) = S \iff S$ is geodesic-convex
- $I_{mo}(S) = S \iff S$ is monophonic-convex
- $I_{m^3}(S) = S \iff S$ is m^3 -convex

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Convexities (P_3 , geodesic, monophonic, m^3)

- $I_{P_3}(S) = S \iff S$ is convex in the P_3 -convexity
- $I_{geo}(S) = S \iff S$ is convex in the geodesic convexity
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Convexities (P_3 , geodesic, monophonic, m^3) Several papers:

- P₃-convexity [Barbosa et al., 2012, SIAM DM]
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Convexity (P_3^*)

▶
$$I_{P_3^*}(S) = S \cup \{ \text{induced } P_3 \text{'s between vert. of } S \}$$

Convexity of induced paths of order 3

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Convexity (P_3^*)

- ▶ $I_{P_3^*}(S) = S \cup \{ \text{induced } P_3 \text{'s between vert. of } S \}$
- ► $I_{mo}(S) = I_{m^3}(S) \cup I_{P_3^*}(S)$

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Convexity (P_3^*)

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•
$$I_{mo}(S) = I_{m^3}(S) \cup I_{P_3^*}(S)$$

• $I_{P_3^*}(S) = I_{P_3}(S)$, if G is triangle-free

Convexity of induced paths of order 3

Some definitions

▶ $I^{k}[\cdot]$ is the k-th iterate of the interval function $I[\cdot]$

Convexity parameters

Convexity of induced paths of order 3

Convexity in graphs P₃-hull number P₃-convexity number

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Some definitions

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• The percolation time of S is
$$t(S) = \min_k \left\{ I^k(S) = I^{k+1}(S) \right\}$$

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• The hull of S is $hull(S) = I^{t(S)}(S)$

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if $hull(S) = V(G)$

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Convexity parameters

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- ► *S* is an interval set

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Convexity parameters

• hull number hn(G): $\min_{S} |S|$

 $\min_{S} |S|$ s.t. S is a hull set

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if $hull(S) = V(G)$
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Convexity parameters

- hull number hn(G):
- interval number in(G):
- convexity number cx(n):
- $\min_{S} |S|$ s.t. S is a hull set
- $\min_{S} |S|$ s.t. S is an interval set
- $\max_{S} |S|$ s.t. $S \neq V(G)$ is convex

Convexity of induced paths of order 3

Convexity in graphs

Some definitions

- ▶ $I^k[\cdot]$ is the *k*-th iterate of the interval function $I[\cdot]$
- The percolation time of S is $t(S) = \min_k \left\{ I^k(S) = I^{k+1}(S) \right\}$
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Convexity parameters

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 $\min_{S} |S|$ s.t. *S* is a hull set $\min_{S} |S|$ s.t. *S* is an interval set $\max_{S} |S|$ s.t. $S \neq V(G)$ is convex $\max_{S} t(S)$ s.t. *S* is a hull set Convexity of induced paths of order 3

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Convexity parameters

- hull number hn(G):
- interval number in(G):
- convexity number cx(n):
- percolation time t(G):
- Carathéodory no. cth(G):

 $\max_{S} |S|$ s.t. $hull(S) \neq \bigcup_{x \in S} hull(S-x)$

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Convexity of induced paths of order 3

- $\min_{S} |S|$ s.t. S is a hull set
- $\min_{S} |S|$ s.t. S is an interval set
- : max_S |S| s.t. $S \neq V(G)$ is convex
- G: $\max_{S} t(S)$ s.t. S is a hull set

P₃ convexity (NP-hard results)

Geodesic convexity (NP-hard results)

Convexity of induced paths of order 3

Convexity in graphs P₃-hull number P₃-convexity number

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P₃ convexity (NP-hard results)

▶ Hull number *hn*(*G*) [Centeno et al., 2011, TCS]

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Convexity of induced paths of order 3 Convexity in graphs

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- ▶ Percolation time $t(G) \ge 4$? [Benevides et al., 2013, sub]

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Convexity of induced paths of order 3

Convexity in graphs P₃-hull number P₃-convexity number

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- ▶ Percolation time t(G) ???
- Carathéodory number cth(G) [Dourado et al., 2013, sub]

Convexity of induced paths of order 3

*P*₃ convexity (NP-hard results) (bipartite graphs)

- ▶ Hull number *hn*(*G*) [Centeno et al., 2011, TCS]
- ▶ Interval number *in*(*G*) [Centeno et al., 2009, ENDM]
- ► Convexity number cx(G) [Centeno et al., 2009, ENDM]
- ▶ Percolation time $t(G) \ge 9$? [Benevides et al., 2013, sub]
- ► Carathéodory no. *cth*(*G*) [Barbosa et al., 2012, SIAM J.DM]

Geodesic convexity (NP-hard results)

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- ▶ Interval number *in*(*G*) [Dourado et al., 2010, DM]
- ► Convexity number cx(G) [Dourado et al., 2012, G&C]
- ▶ Percolation time *t*(*G*) ???
- ► Carathéodory number *cth*(*G*) [Dourado et al., 2013, sub]

Convexity of induced paths of order 3

P₃-hull number is NP-hard in bipartite graphs

Decision Problem $(hn_{P_3}(G) \le k?)$

Given a graph G and an integer k, the P_3 -hull number $hn_{P_3}(G)$ is at most k?

Reduction from SAT with the following restrictions:

- Every clause has at most 3 variables,
- Every literal is in some clause and,
- ► For every variable x_i, there are at most 3 clauses containing either x_i or x_i.

Example

$$\Phi = (x_1 \vee x_2) \land (x_1 \vee \overline{x_2}) \land (\overline{x_1} \vee x_2)$$

Convexity of induced paths of order 3

Convexity in graphs **P3-hull number** P3-convexity number

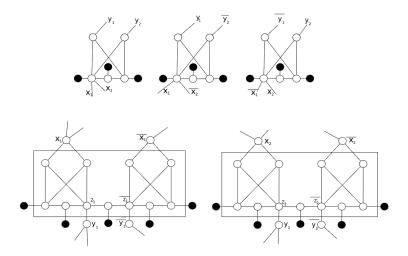


Figura : Reduction from SAT: $\Phi = (x_1 \lor x_2) \land (x_1 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2)$

Convexity of induced paths of order 3

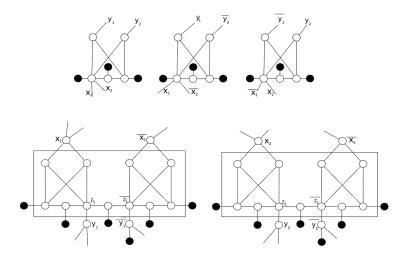


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Convexity of induced paths of order 3

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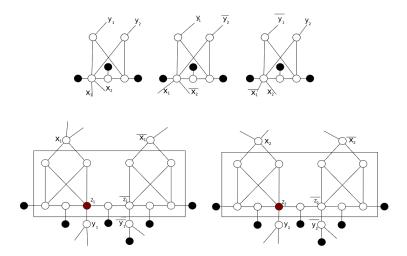


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Convexity of induced paths of order 3

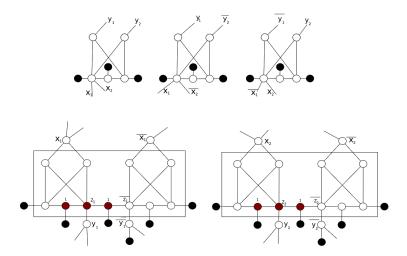


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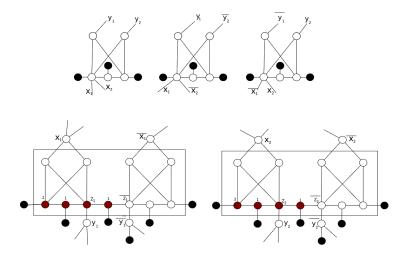


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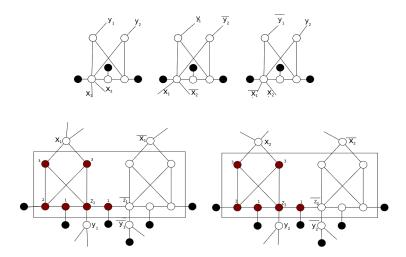


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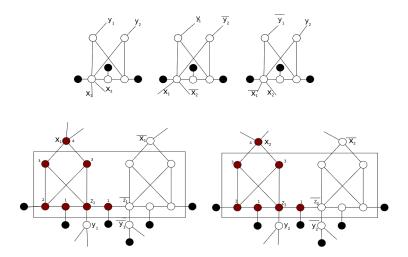


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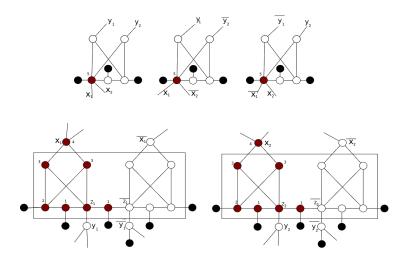


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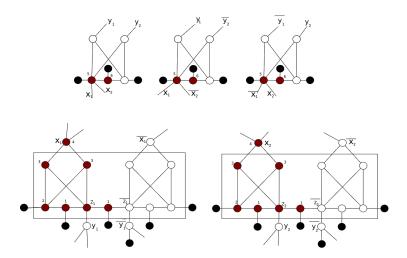


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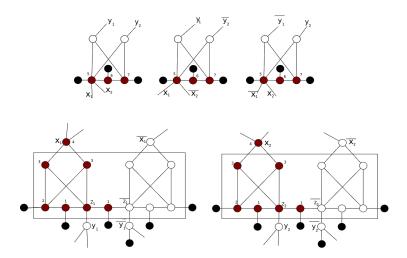


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Convexity of induced paths of order 3

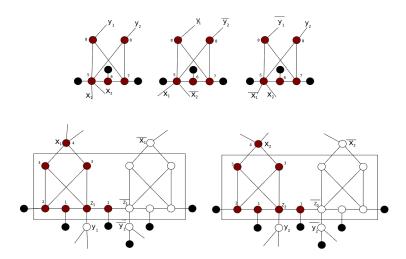
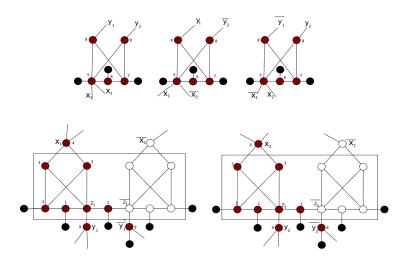


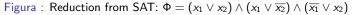
Figura : Reduction from SAT: $\Phi = (x_1 \lor x_2) \land (x_1 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2)$

Convexity of induced paths of order 3

Convexity in graphs P₃-hull number P₃-convexity number

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Convexity of induced paths of order 3

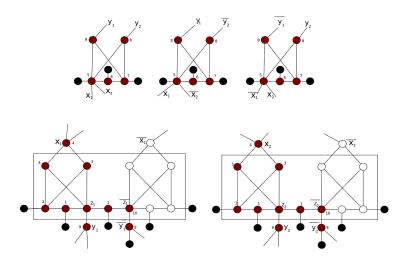


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Convexity of induced paths of order 3

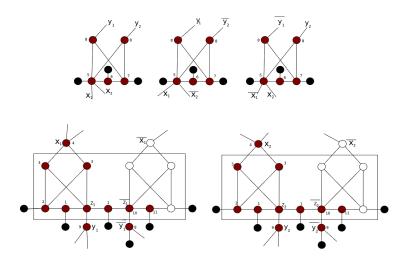


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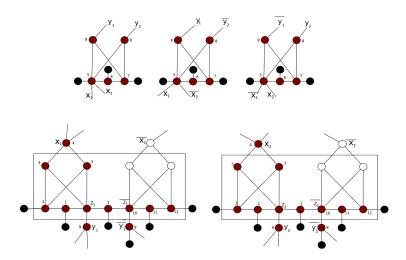
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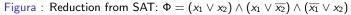
Convexity of induced

paths of order 3

P3-hull number

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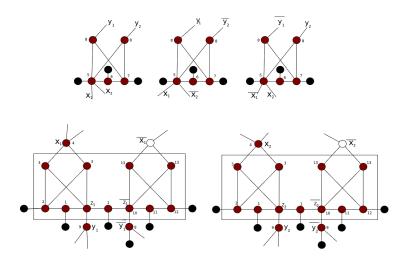
Convexity in graphs P3-hull number

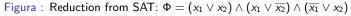
Convexity of induced

paths of order 3

3-convexity number

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Convexity of induced paths of order 3

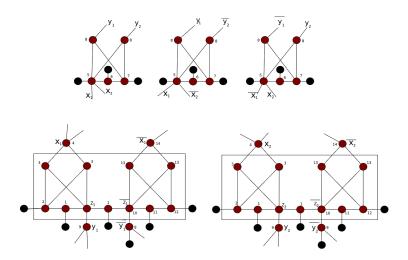


Figura : Reduction from SAT: $\Phi = (x_1 \lor x_2) \land (x_1 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2)$

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Convexity of induced paths of order 3

Decision Problem $(cx_{P_3}(G) \ge k?)$

Given a graph G and an integer k, the P_3 -convexity number $cx_{P_3}(G)$ is at least k?

Reduction from SET-PACKING

Given integers k and m, and m sets S_1, \ldots, S_m , does there exist k pairwise disjoint sets?

Example (k = 3, m = 5)

► $S_1 = \{a, b, c\},$	$S_2 = \{b, f, g\}$
► $S_3 = \{d, e, f\},$	$S_4 = \{c, e, g\}$

•
$$S_5 = \{g, h, i\}$$

Convexity of induced paths of order 3

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Convexity of induced paths of order 3

Convexity in graphs P₃-hull number P₃-convexity number

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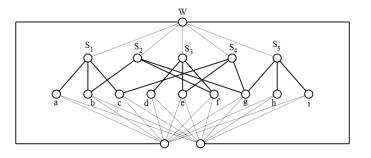


Figura : Reduction from SET-PACKING

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Convexity of induced paths of order 3

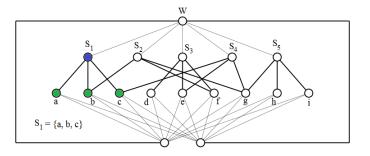


Figura : Reduction from SET-PACKING

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Convexity of induced paths of order 3

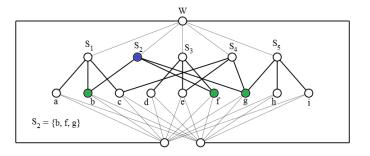


Figura : Reduction from SET-PACKING

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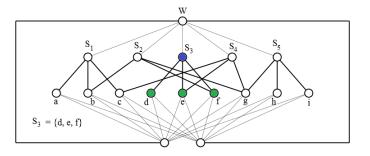


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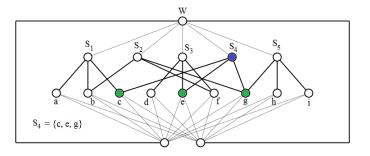


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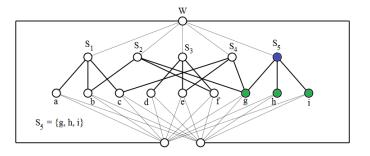


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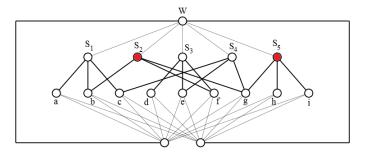


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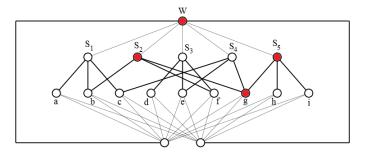


Figura : Reduction from SET-PACKING

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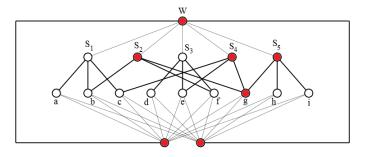


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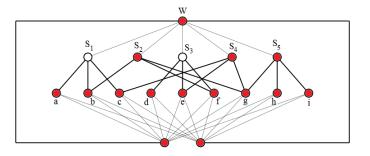


Figura : Reduction from SET-PACKING

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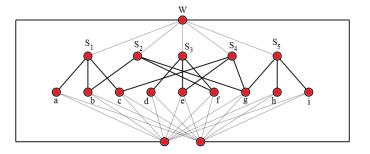


Figura : Reduction from SET-PACKING

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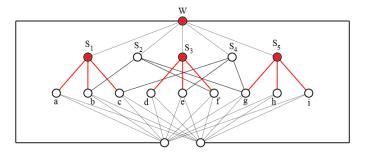


Figura : Reduction from SET-PACKING

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