Convexity of induced paths of order 3

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This is a joint work with
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Jayme Szwarcfiter (UFRJ, Rio de Janeiro, Brazil)

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Convexity in Graphs

Intervals (P_3, geodesic, monophonic, m^3)

Given a graph G and a set S of vertices, let:

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Convexities (P_3, geodesic, monophonic, m^3)

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- $I_{P_3^*}(S) = I_{P_3}(S)$, if G is triangle-free
- ...

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- Carathéodory no. $cth(G)$:

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Complexity results

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- Percolation time $t(G) \geq 9$? [Benevides et al., 2013, sub]
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P_3-hull number is NP-hard in bipartite graphs

Decision Problem ($hn_{P_3}(G) \leq k$?)

Given a graph G and an integer k,
the P_3-hull number $hn_{P_3}(G)$ is at most k ?

Reduction from SAT with the following restrictions:

- Every clause has at most 3 variables,
- Every literal is in some clause and,
- For every variable x_i, there are at most 3 clauses containing either x_i or $\overline{x_i}$.

Example

$\Phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2)$

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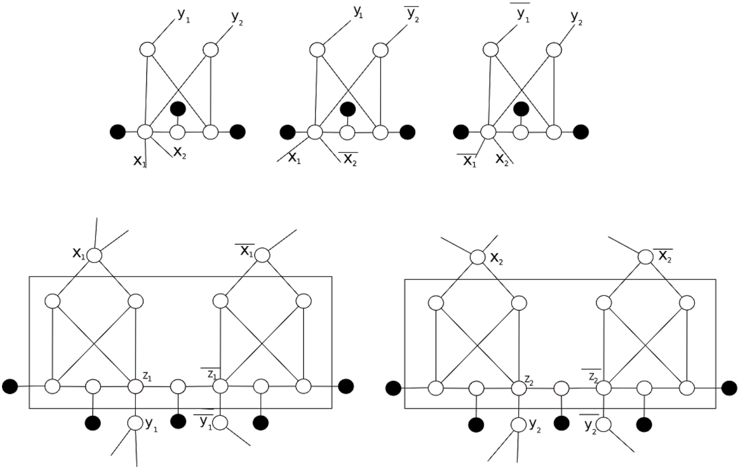

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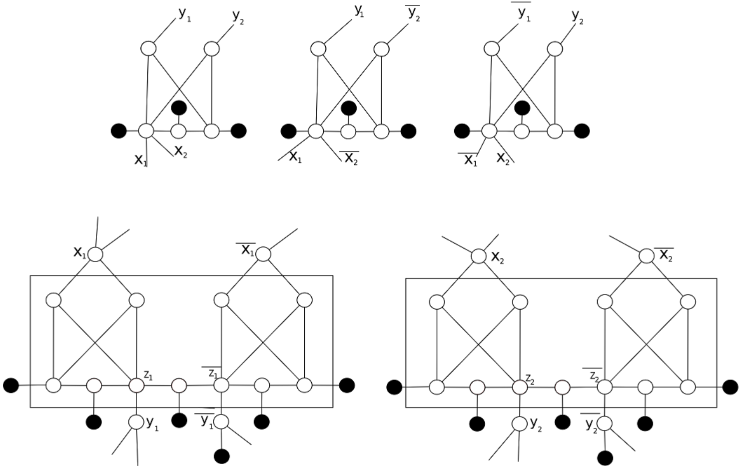

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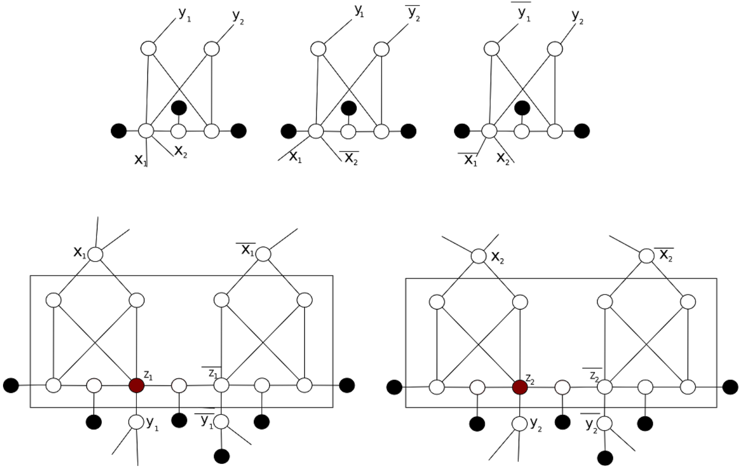

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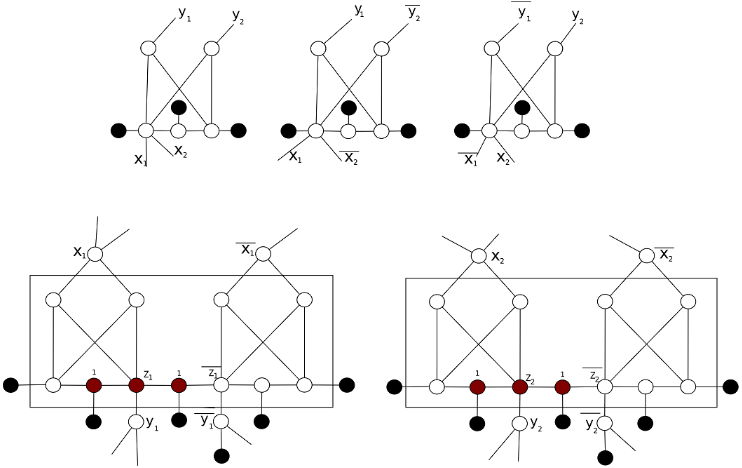

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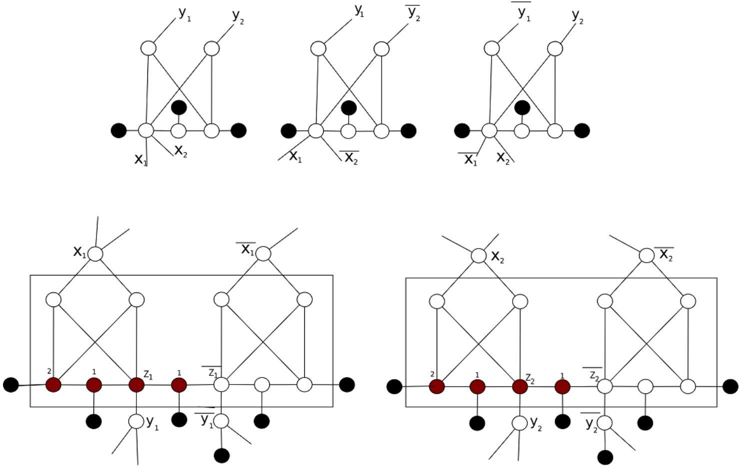

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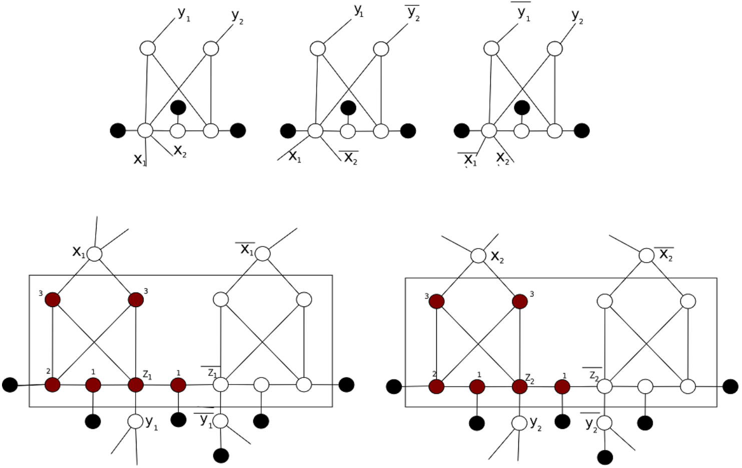

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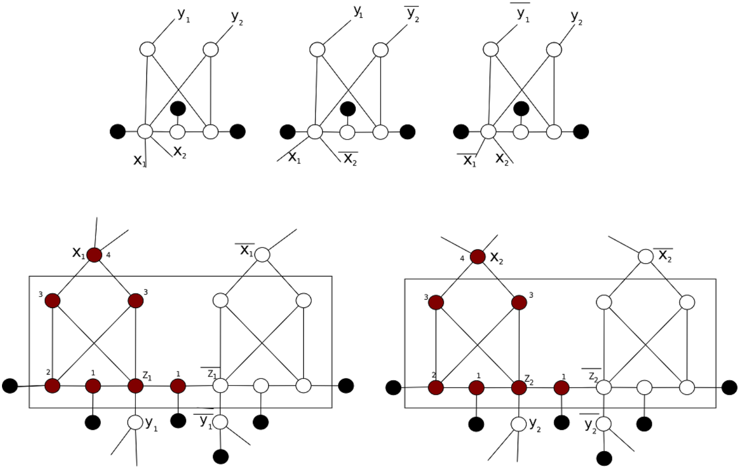

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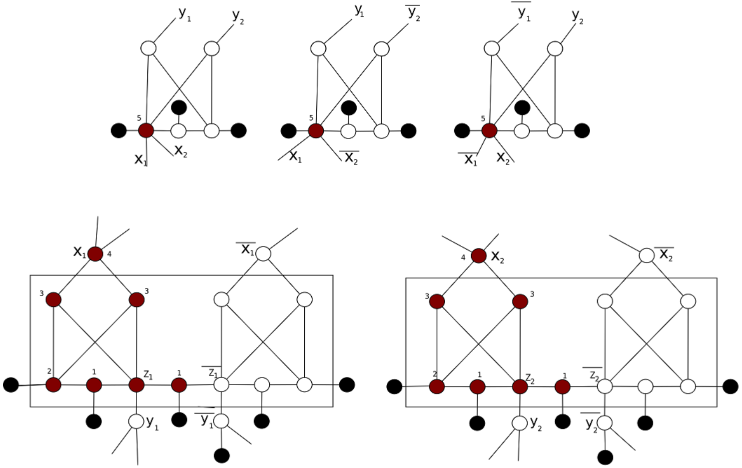

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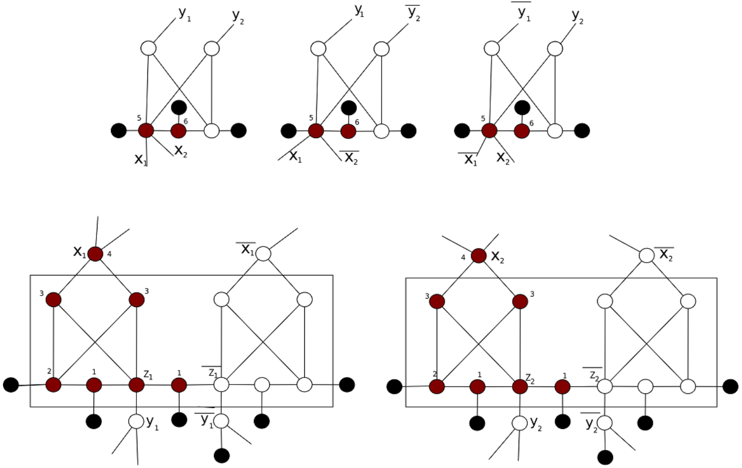

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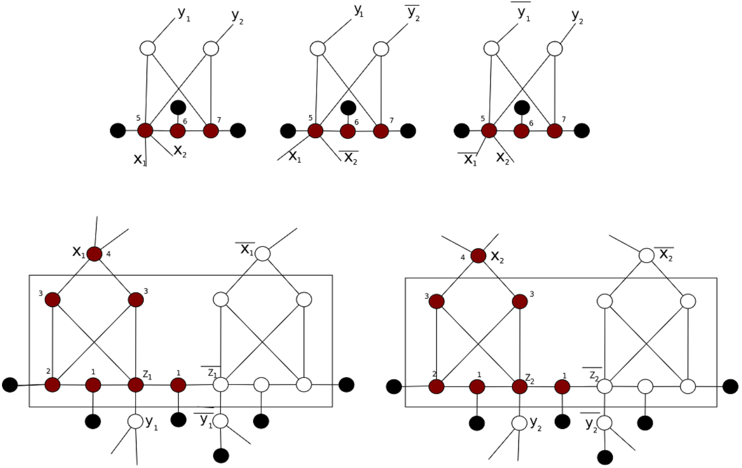

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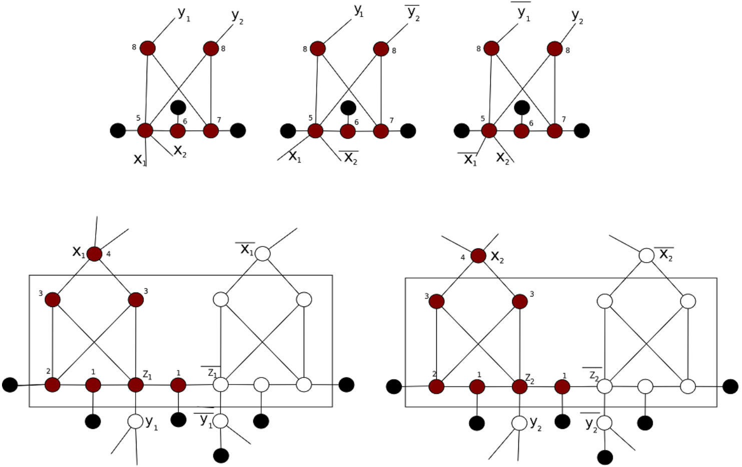

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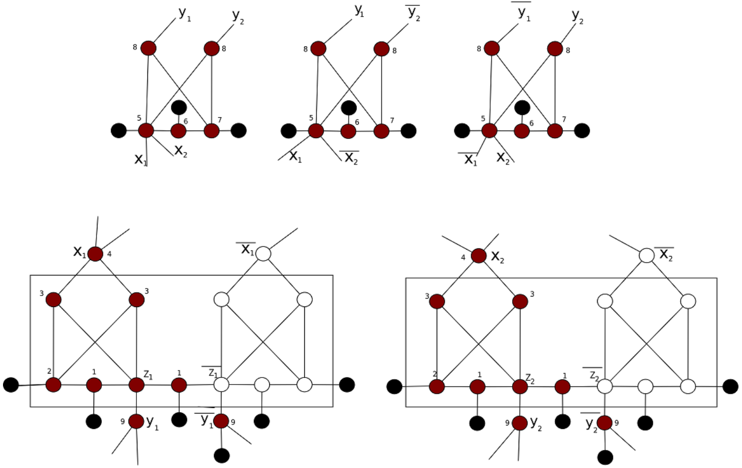

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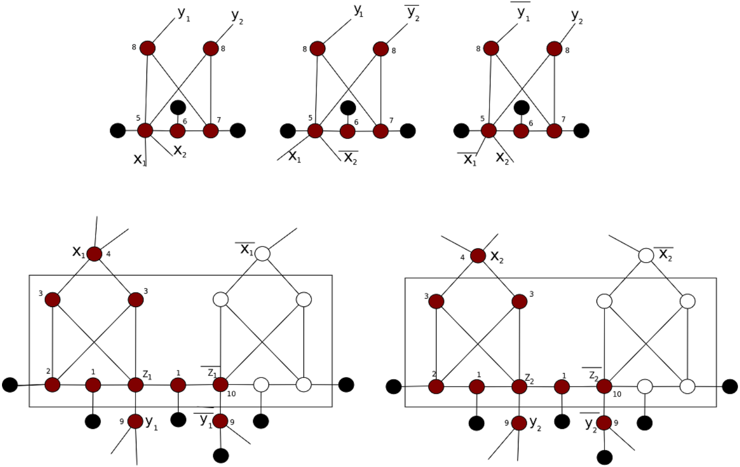

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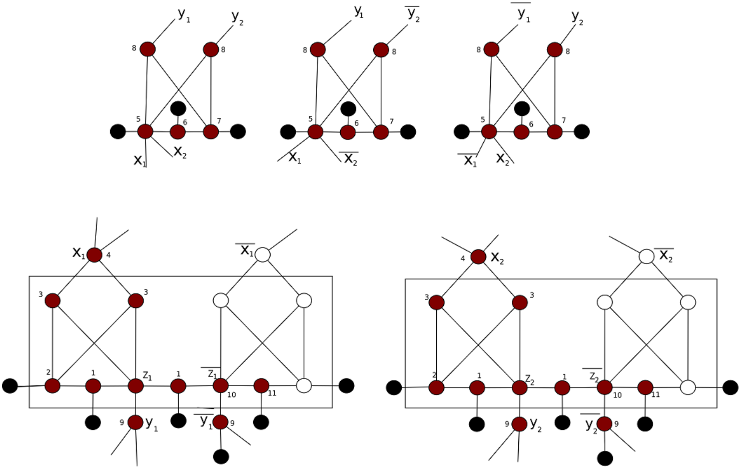

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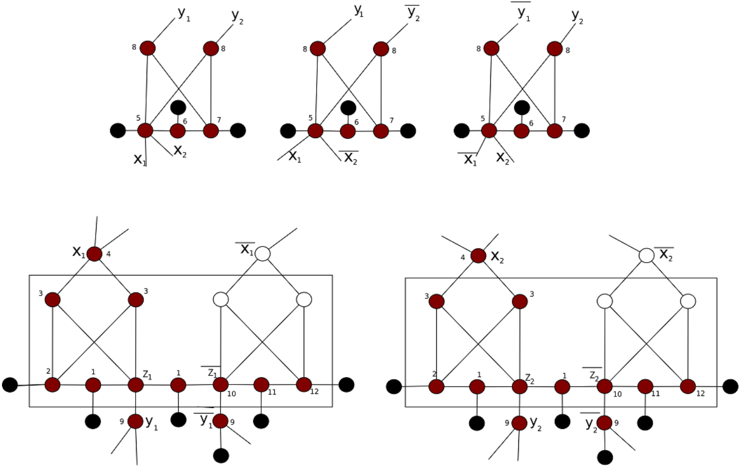

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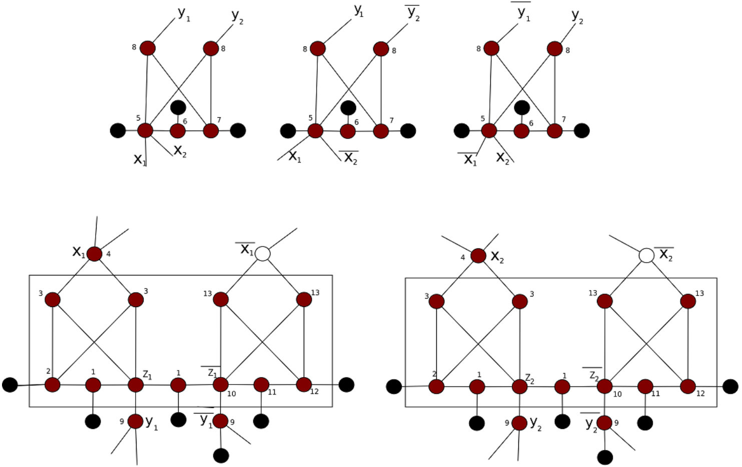

Figura : Reduction from SAT: $\Phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2)$

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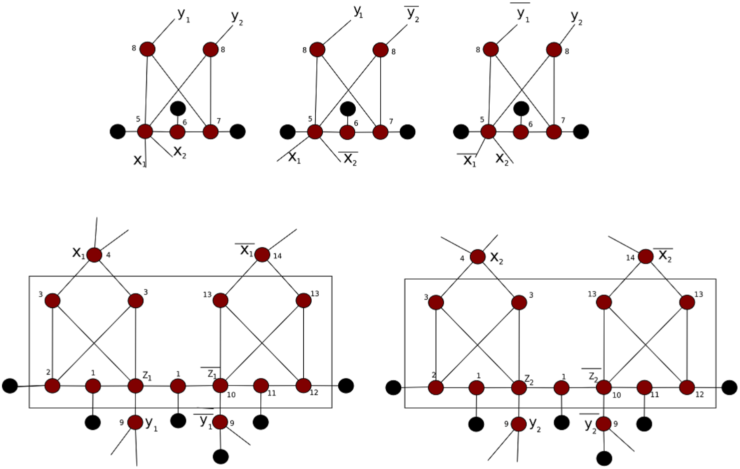

Figura : Reduction from SAT: $\Phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2)$

P_3-convexity no. is NP-hard in bipartite graphs

Decision Problem ($cx_{P_3}(G) \geq k$?)

Given a graph G and an integer k,
the P_3-convexity number $cx_{P_3}(G)$ is at least k ?

Reduction from SET-PACKING

Given integers k and m, and m sets $S_1, \ldots, S_m$,
does there exist k pairwise disjoint sets?

Example ($k = 3,\ m = 5$)

- $S_1 = \{a, b, c\}$, $S_2 = \{b, f, g\}$
- $S_3 = \{d, e, f\}$, $S_4 = \{c, e, g\}$
- $S_5 = \{g, h, i\}$

P_3-convexity no. is NP-hard in bipartite graphs

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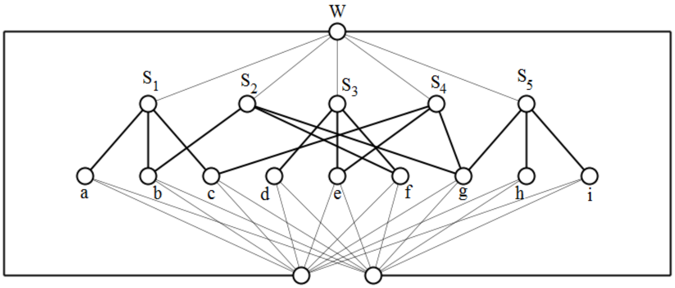


Figura : Reduction from SET-PACKING

P_3-convexity no. is NP-hard in bipartite graphs

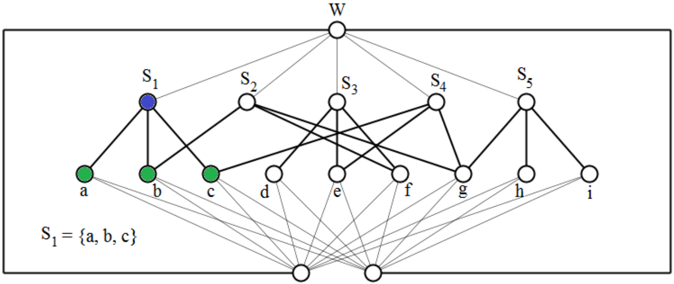


Figura : Reduction from SET-PACKING

P_3-convexity no. is NP-hard in bipartite graphs

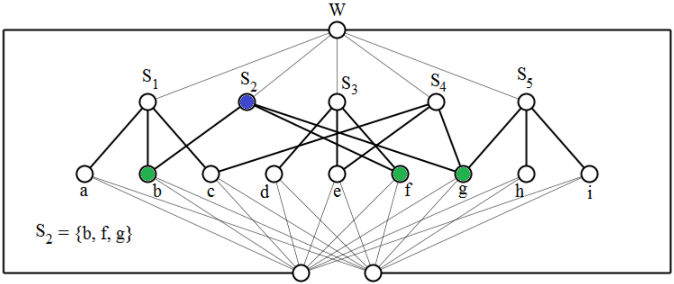
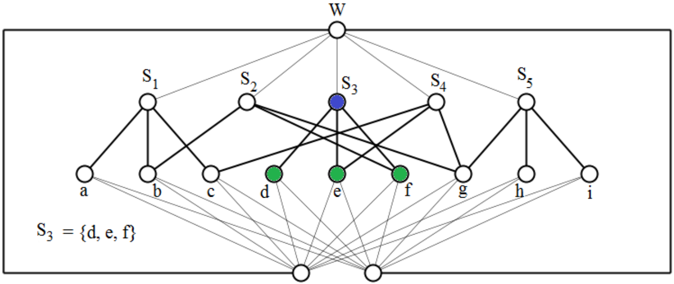


Figura : Reduction from SET-PACKING

P_3-convexity no. is NP-hard in bipartite graphs



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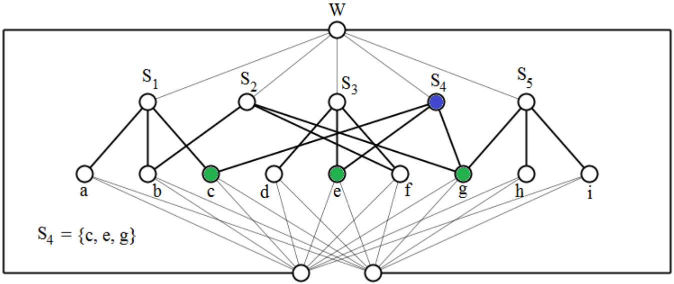

Figura : Reduction from SET-PACKING

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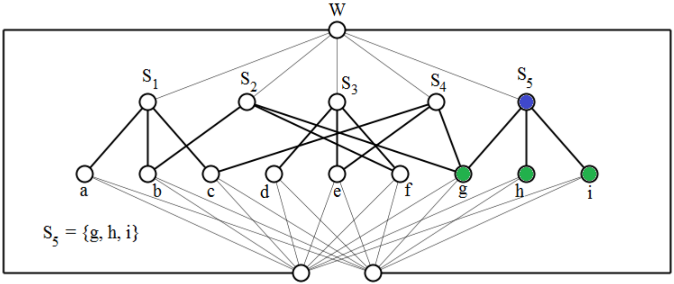

Figura : Reduction from SET-PACKING

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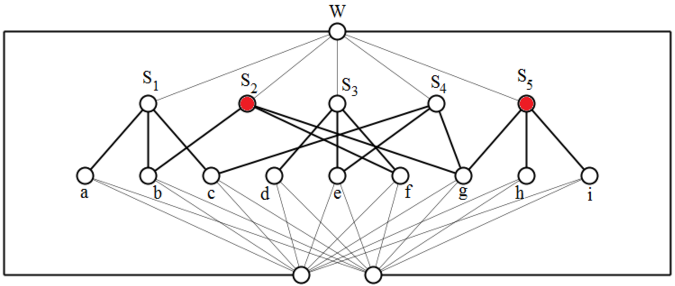

Figura : Reduction from SET-PACKING

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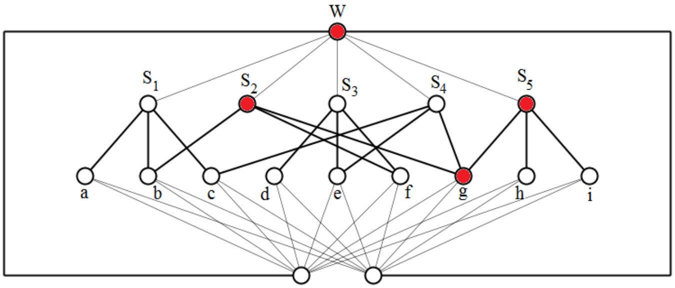

Figura : Reduction from SET-PACKING

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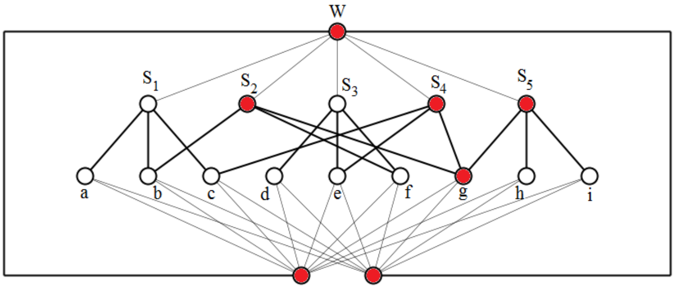

Figura : Reduction from SET-PACKING

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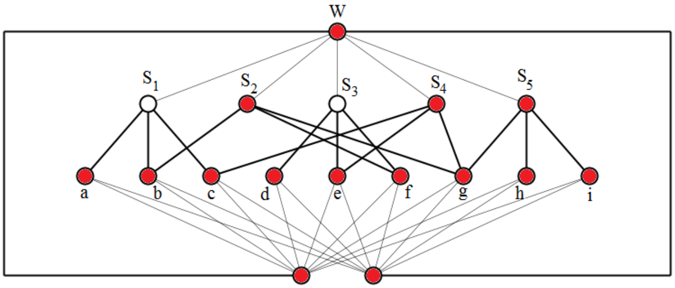

Figura : Reduction from SET-PACKING

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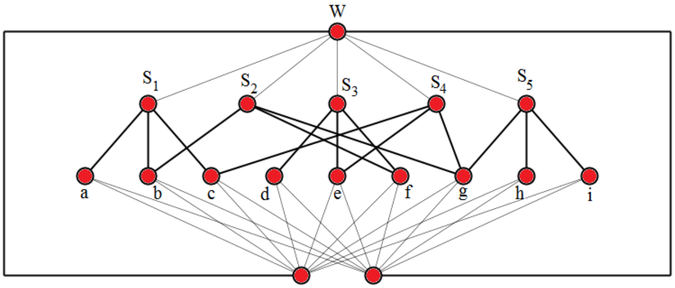

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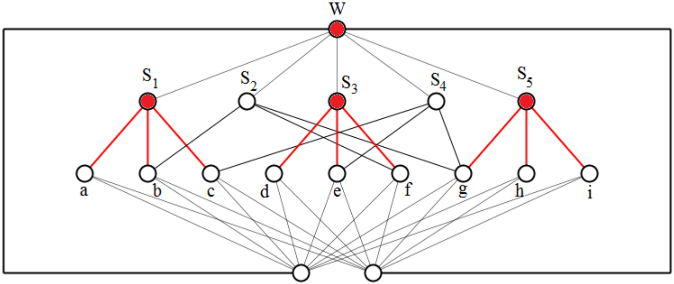

Figura : Reduction from SET-PACKING