A note on permutation regularity

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This is joint work with

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Szemerédi Regularity Lemma

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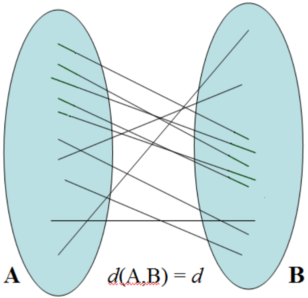
Extensions: hypergraphs, permutations, ...

ε-regular pairs

A pair (A, B) is ε-regular if, for every pair (A', B')

$$\substack{A' \subseteq A \\ B' \subseteq B}, \text{ s.t. } \substack{|A'| \geq \varepsilon |A| \\ |B'| \geq \varepsilon |B|} \quad \Rightarrow \quad d(A', B') \ = \ d(A, B) \pm \varepsilon$$

where $d(A, B) = e(A, B)/|A||B|$.



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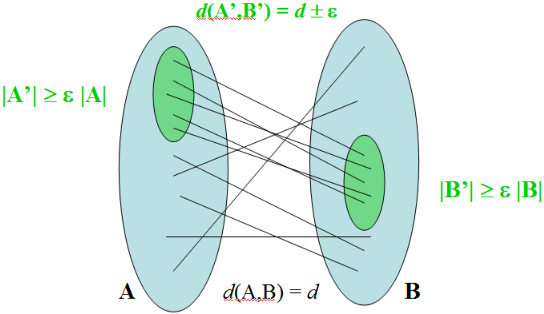

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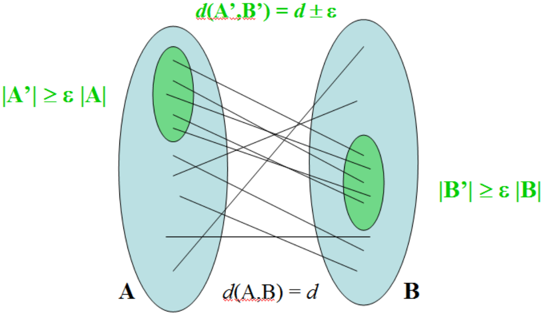

Obs: $\varepsilon \to 0 \Rightarrow$ ε-regular pair "close" to random bipartite graph.

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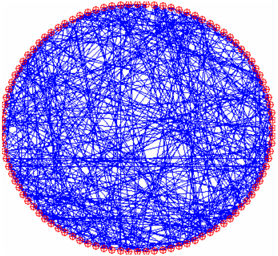

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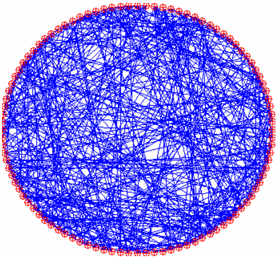

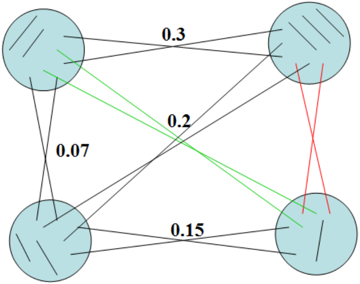

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$|V_1| = \ldots = |V_k|$ with an exceptional set V_0 with $\leq \varepsilon n$ vertices

Or uniform partition
It is very easy to remove that exceptional set in the graph case

$$\left\lfloor \frac{n}{k} \right\rfloor \leq |V_1| \leq \ldots \leq |V_k| \leq \left\lceil \frac{n}{k} \right\rceil$$

Permutation Regularity Lemma

A permutation σ on $[n] = \{1, 2, \ldots, n\}$ is a bijective function of the set $[n]$ into itself.

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Applications: quasirandomness, counting subpermutations,...

Encoding permutations

A permutation σ on $[n]$ can be encoded as a bipartite graph G_σ whose color classes A and B are disjoint copies of $[n]$, and where $(a, b) \in A \times B$ is an edge if and only if $\sigma(a) < b$.

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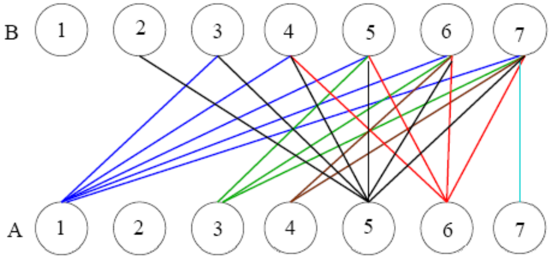
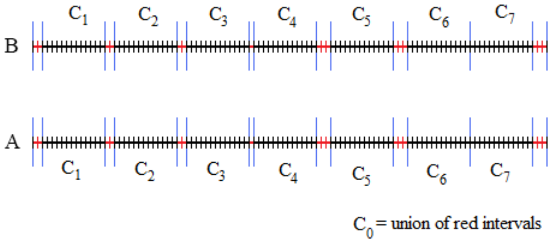
$$\sigma = (2, 7, 4, 5, 1, 3, 6)$$



Partition into intervals

Cooper's partition is into intervals and the proof is only for equitable partitions (with an exceptional non-interval set C_0)



Partition into intervals

A pair of intervals (C_a, C_b) is **ε-regular** if, for every pair (I_a, I_b) of subintervals where

$$\substack{I_a \subseteq C_a \\ I_b \subseteq C_b}, \text{ s.t. } \substack{|I_a| \geq \varepsilon |C_a| \\ |I_b| \geq \varepsilon |C_b|} \quad \Rightarrow \quad d(I_a, I_b) \ = \ d(C_a, C_b) \pm \varepsilon$$

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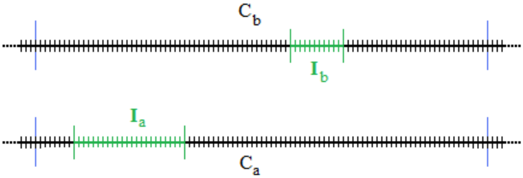

$$\mathbb{P}\left(\substack{\sigma(x)<y: \\ (x,y)\in I_a \times I_b}\right) \ = \ \mathbb{P}\left(\substack{\sigma(x)<y: \\ (x,y)\in C_a \times C_b}\right) \ \pm \ \varepsilon$$

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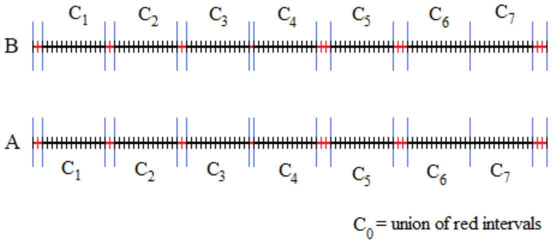

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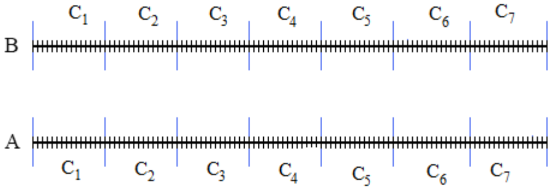

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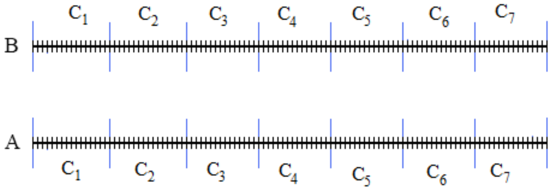

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Our main result

The Permutation Regularity Lemma can also return a uniform k-partition.

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And more: For this value of k, all uniform partitions with k intervals satisfy the regularity lemma.

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Theorem (Hoppen, Kohayakawa, Sampaio, 2009)
For every $\varepsilon > 0$ and $m > 1$
there **exist** $M > m$ and n_0 s.t.
For every permutation σ on $[n]$ with $n > n_0$
there **exists** a constant $k \in [m, M]$ s.t.
Every uniform k-partition $P = (C_i)_{i=1}^k$ of σ is ε-regular. (**Our result**)

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Proof ideas:

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- We obtain a sharper inequality that helps us to change a little bit the interval sizes in the regularization process.
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Importance and applications:

- Partitions are more beautiful and the exceptional set was removed forever. :-)
- Now it is possible to apply successive regularizations that refine each other, like in the seminal paper "*Limits of dense graph sequences*" of Lovász and Szegedy.
- We use a weaker version in another paper "*Limits of permutation sequences*".

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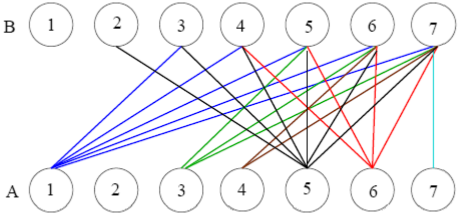

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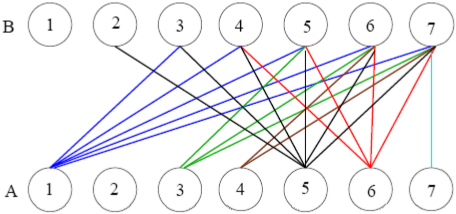

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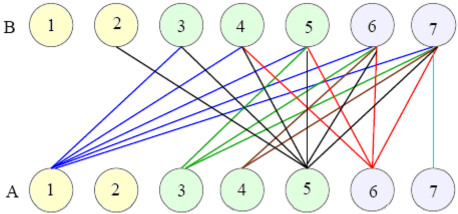

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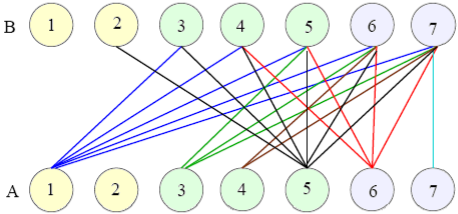

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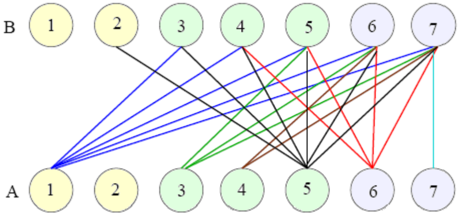

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The lines sum $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \leq \begin{bmatrix} 9/4 \\ 23/18 \\ 1/6 \end{bmatrix} < \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

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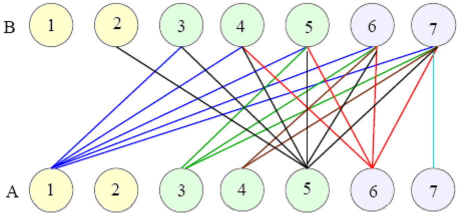

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"Weighted permutation"

The lines sum $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \leq \begin{bmatrix} 9/4 \\ 23/18 \\ 1/6 \end{bmatrix} < \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

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Moreover, given weighted permutations Q_1 and Q_2 on $[n]$, we have

$$|t(\tau, Q_1) - t(\tau, Q_2)| \leq 2m^2 \cdot d_\square(Q_1, Q_2)$$

A note on permutation regularity

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