# Graph convexity partizan games

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**Future papers:** Partizan convexity games
Convex Forming game
Coloring game in grids
Harmonious Coloring game

2. doi> ARAÚJO, SAMUEL N. ; BRITO, JOÃO MARCOS ; FOLZ, RAQUEL ; DE FREITAS, ROSIANE ; **Sampaio, Rudini M.** . Graph convexity impartial games: complexity and winning strategies. THEORETICAL COMPUTER SCIENCE JCR, v. 998, p. 114534, 2024.

4. doi> CHANDRAN S. V., ULLAS ; KLAV'AR, SANDI ; P. K., NEETHU ; **Sampaio, Rudini** . The general position avoidance game and hardness of general position games. THEORETICAL COMPUTER SCIENCE JCR, v. 988, p. 114370, 2024. Citações: WEB OF SCIENCE™ 2 | SCOPUS 1

8. doi> LIMA, CARLOS V.G.C. ; MARCILON, THIAGO ; MARTINS, NICOLAS ; **Sampaio, Rudini** . The connected greedy coloring game. THEORETICAL COMPUTER SCIENCE JCR, v. 940, p. 1-13, 2023. Citações: WEB OF SCIENCE™ 1 | SCOPUS 2

9. doi> LIMA, CARLOS V.G.C. ; MARCILON, THIAGO ; MARTINS, NICOLAS ; **Sampaio, Rudini** . PSPACE-hardness of variants of the graph coloring game. THEORETICAL COMPUTER SCIENCE JCR, v. 909, p. 87-96, 2022. Citações: WEB OF SCIENCE™ 2 | SCOPUS 3

10. doi> COSTA, EURINARDO RODRIGUES ; MARTINS, NICOLAS ALMEIDA ; **Sampaio, Rudini** . Spy game: FPT-algorithm, hardness and graph products. THEORETICAL COMPUTER SCIENCE JCR, v. 923, p. 304-317, 2022.

11. ☆ doi> COSTA, EURINARDO ; PESSOA, VICTOR LAGE ; **Sampaio, Rudini** ; SOARES, RONAN . PSPACE-completeness of two graph coloring games. THEORETICAL COMPUTER SCIENCE JCR, v. 824, p. 36-45, 2020. Citações: WEB OF SCIENCE™ 4 | SCOPUS 6

27. doi> COHEN, NATHANN ; A.MARTINS, NÍCOLAS ; MC INERNEY, FIONN ; NISSE, NICOLAS ; PÉRENNES, STÉPHANE ; **Sampaio, Rudini** . Spy-Game on graphs: Complexity and simple topologies. THEORETICAL COMPUTER SCIENCE JCR, v. 725, p. 1-15, 2017. Citações: WEB OF SCIENCE™ 6 | SCOPUS 9

# Example of a game: Clique Forming Game / Node Kayles (1978)

Alice and Bob select vertices and the set of selected vertices must be a clique.

**Normal game:** the last to play wins.

**Misère game:** the last to play loses.
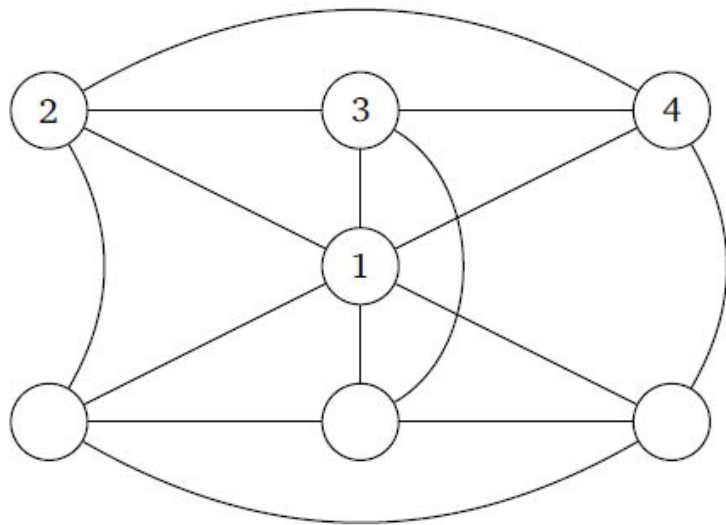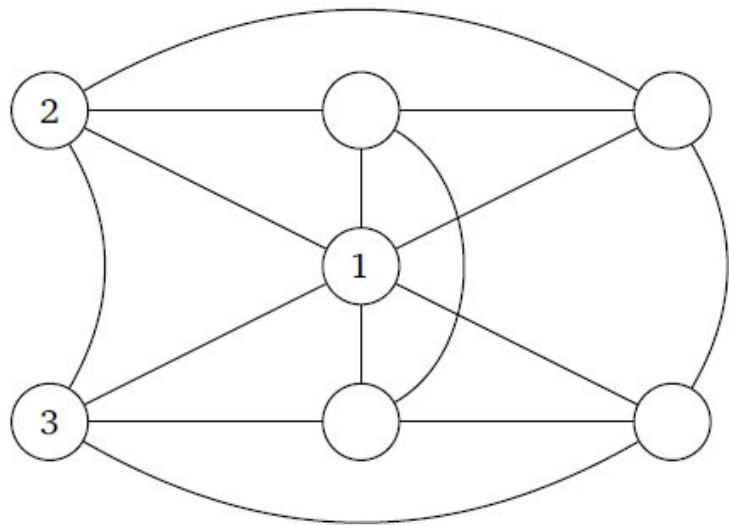
# Graph convexity games

1981 (F. Harary, J. Nieminem): First paper on general graph convexity (in English)

1984 (F. Harary): First graph convexity games.       Set $L$ of labeled vertices.
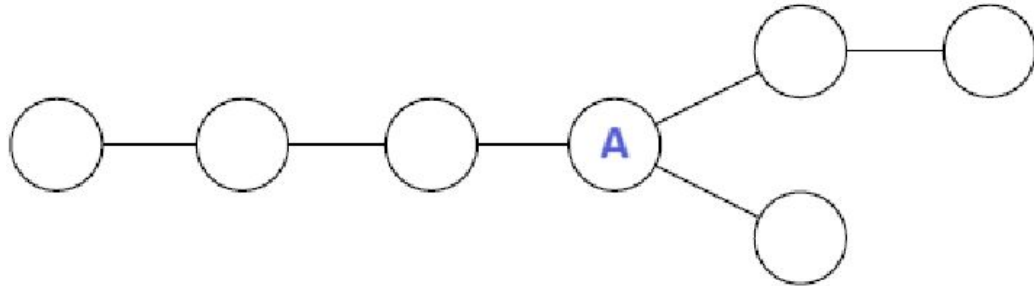
**Interval** $I(L)$**:**   $L$ and any vertex in a shortest path between two vertices of $L$

**Closed Interval Game ($CIG$):** Alice and Bob label unlabeled vertices of a graph that are not in $I(L)$: a shortest path between labeled vertices.
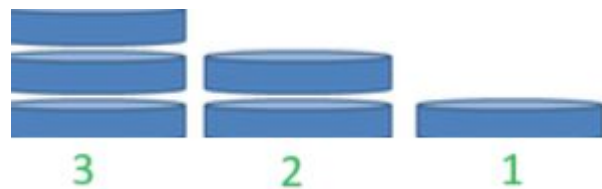
**Impartial game:** There is no forbidden (unlabeled) vertex for a player.

**Normal variant:** the last to play wins

**Misère variant:** the last to play loses
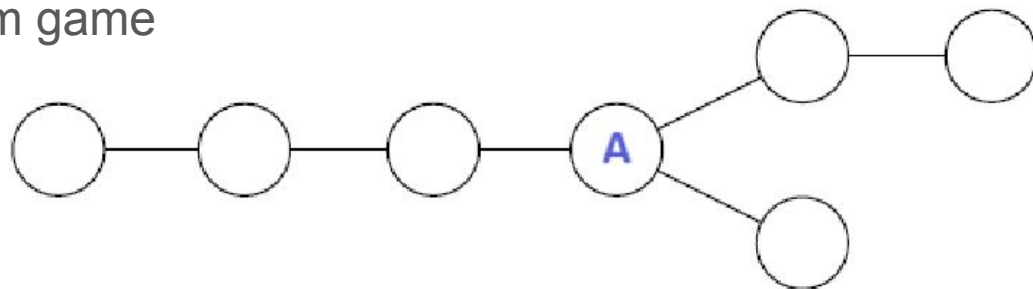
# Impartial Interval Game is poly in trees

**Nim:** Players remove objects from heaps.

**Bouton** (1901) solved Nim
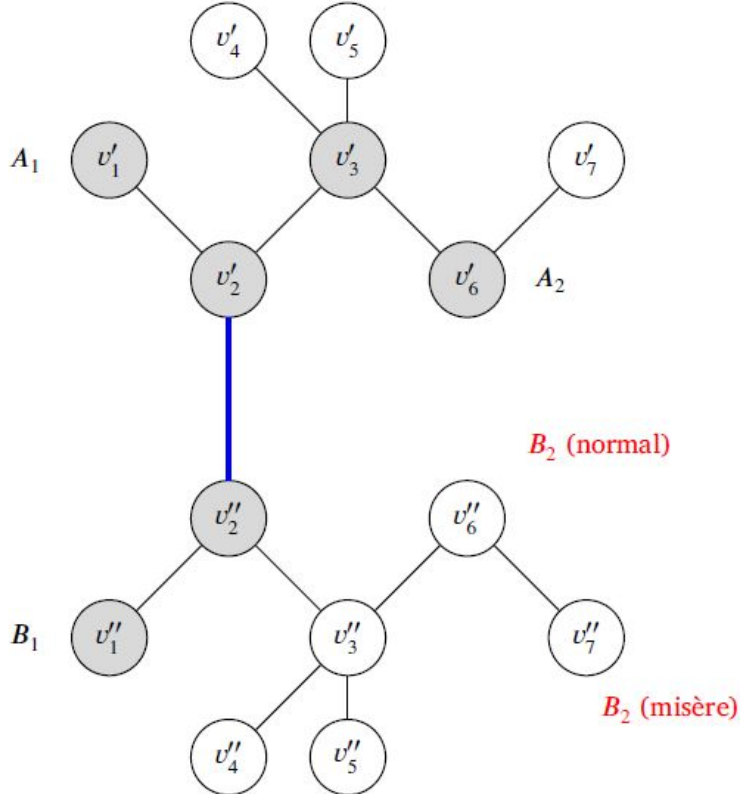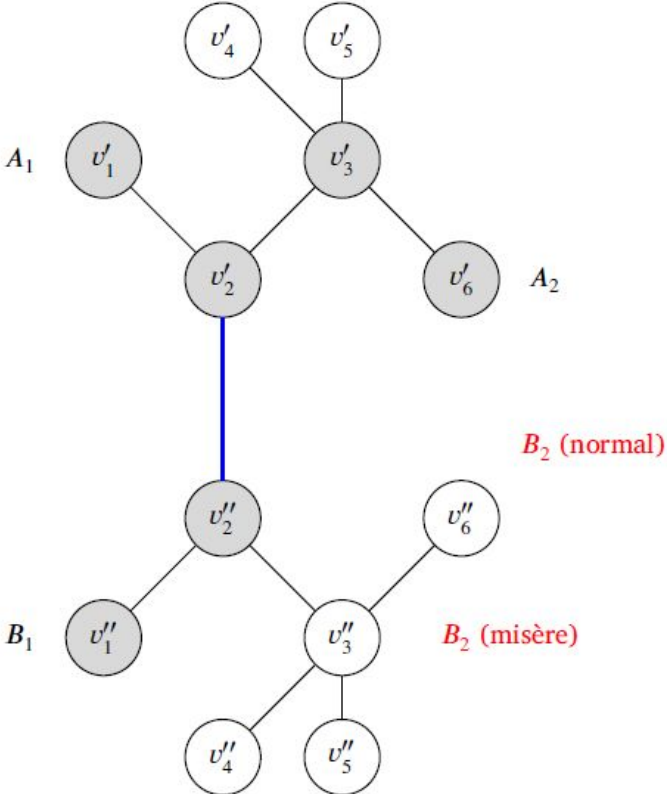
**Sprague-Grundy Theory** (1930s)

relates every impartial game to a Nim game

**Theorem (Araujo, Brito, Folz, Freitas, Sampaio, 2023):**

CIG is polynomial time solvable in trees

# Example of trees in which Bob always wins (normal / misère)

# Partizan Interval Game is poly in trees

**Partizan game:** vertices have owners. Only the owner can label the vertex.

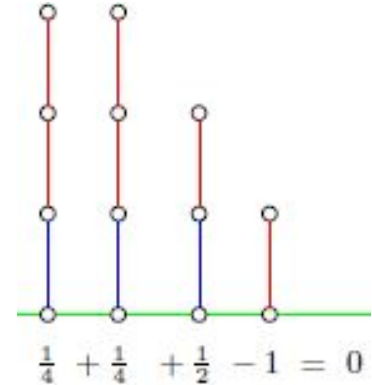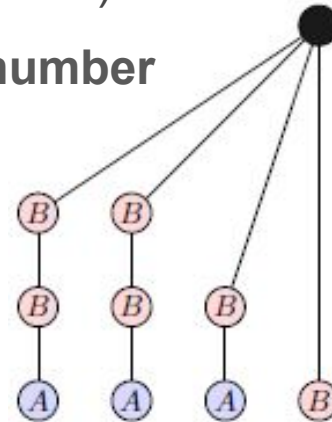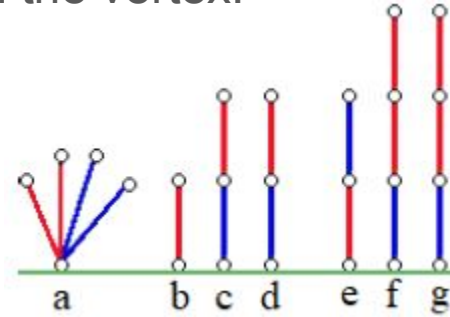**Theorem:** Partizan CIG is polynomial time solvable in trees

Book "*Winning ways for yout mathematical play*"

E. Berlekamp, John Conway, Richard Guy (1982)

**Every position of a partizan game has a number**

**Conway Combinatorial Game Theory**

**Red-Blue Hackenbush game**



$$\tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{2} - 1 = 0$$

# Hull Games are PSPACE-hard

**Closed Hull Game (CHG):** Alice and Bob label unlabeled vertices of a graph that are not in $H(L)$: the convex hull of the labeled vertices.
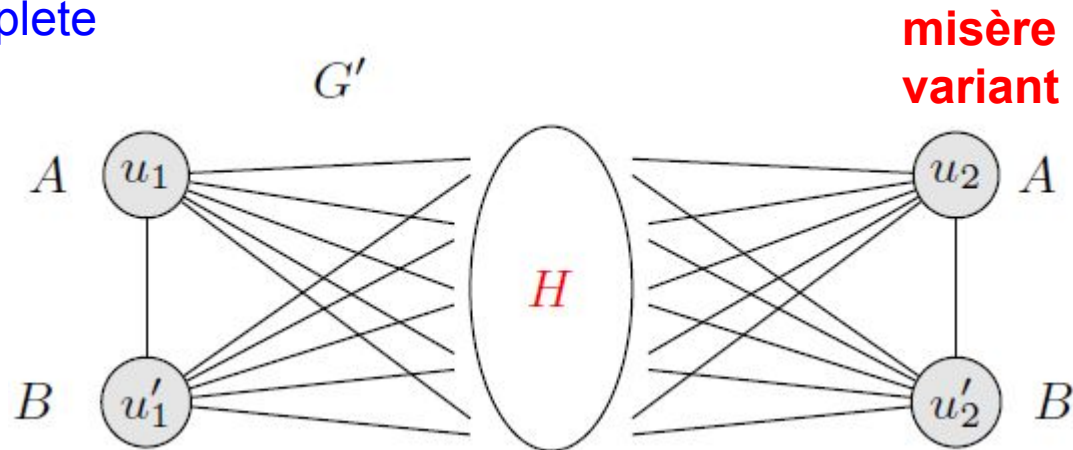
**Convex Hull $H(L)$:** sucessive applications of the interval $I(L)$

Theorem: The normal and misère variants

of Partizan CHG are PSPACE-complete

**misère variant**

Reduction from Partizan

Clique Forming game

1978 (Schaefer): PSPACE-hard

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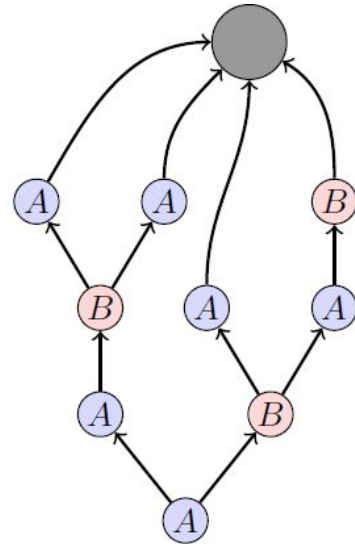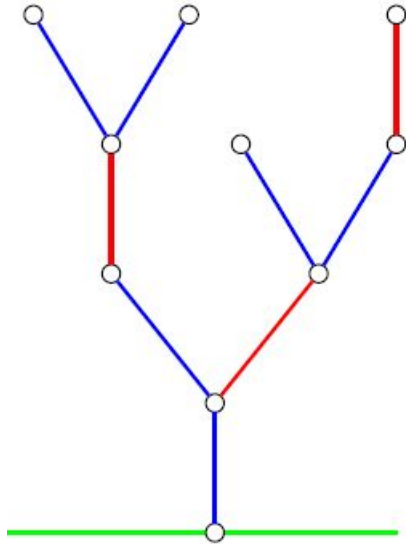
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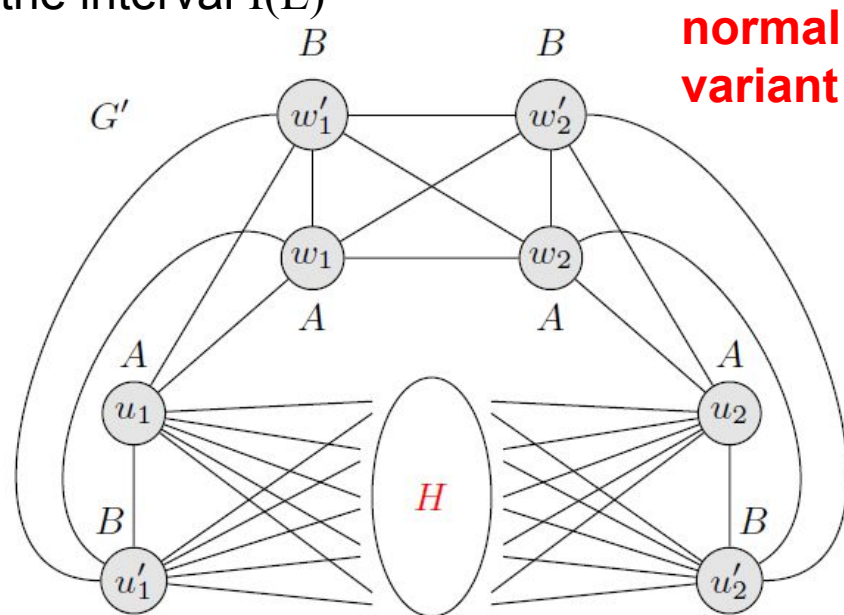
# Other graph convexity games

**Geodesic convexity in tournments [Erdös et al., 1972]:** shortest paths

**Geodesic convexity [Harary, 1981]:** shortest paths (undirected graphs)

**Monophonic convexity [Jamison, 1982]:** induced paths (undirected graphs)

Thanks,
COCOON-2024