Nonrepetitive, acyclic and clique colorings of graphs with few P_4's

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Summary

1. Introduction
2. Primeval and Modular decompositions
3. Disjoint Union, Join and Spiders
4. Coloring (q,q-4)-graphs
5. Split decomposition

Colorings

Definitions

- Proper k-coloring: Every color class induces a stable set.

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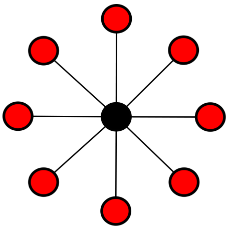

Figure: Star graph

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$$\chi(G) \leq \chi_a(G) \leq \chi_{st}(G) \leq \pi(G) \leq \chi_h(G)$$

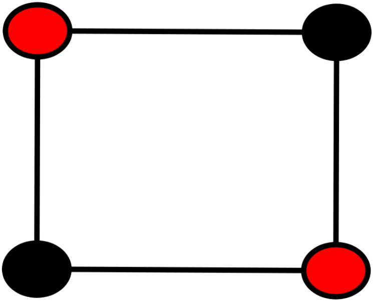

Figure: C_4 with two colors

- A proper coloring, not an acyclic coloring.
- We have a cycle between a pair of color classes.

Figure: P_4 with two colors

- An acyclic coloring, not a star coloring.
- We have a P_4 with only two colors.

Figure: P_6 with three colors

- A star coloring, not a nonrepetitive coloring.
- We have the pattern *white-red-black*.

Figure: P_5 with three colors

- A nonrepetitive coloring, not an harmonic coloring.
- There are two edges betweeen a pair of color classes.

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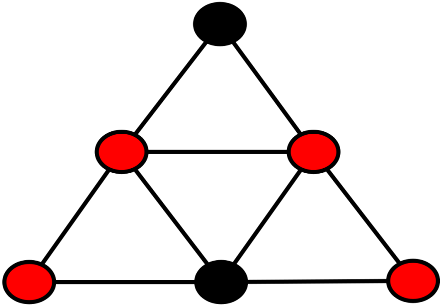
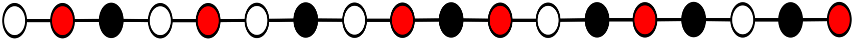


Figure: Graph 2-clique-colorable

Thue's construction

Theorem

- A celebrate theorem of Thue from 1906 asserts that there are arbitrarily long nonrepetitive sequences over the set of just 3 symbols.



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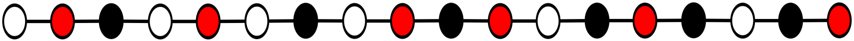

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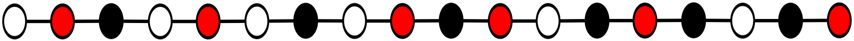

- This implies that $\pi(P_n) = 3$ for every $n > 3$.
- In 2002, Currie proved that

$$\pi(C_n) = \begin{cases} 4, & \text{if } n \in \{5, 7, 9, 10, 14, 17\} \\ 3, & \text{otherwise.} \end{cases}$$

Results about colorings

Acyclic coloring

- $\chi_a(G) \leq 5$ for planar graphs [Borodin, 1979].
- NP-Complete to deciding if $\chi_a(G) \leq 3$ [Kostochka, 1978].
- NP-hard for bipartite graphs [Coleman et al., 1986].

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Star coloring

- NP-hard for planar bipartite graphs [Albertson et al., 2004].

Results about colorings

Non-repetitive coloring

- Co-NP-Complete: determine if a coloring is non-repetitive (even for 4 colors) [Marx and Schaefer, 2009].

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Harmonious coloring

- NP-hard for disconnected cographs [Bodlaender, 1989].
- NP-hard for interval graphs, permutation graphs and split graphs [Asdre et al., 2007].

Results about colorings

Clique coloring

- NP-hard for perfect graphs but polynomial for planar graphs [Kratochvíl and Tuza, 2002].
- In 2004, Bacsó et al. proved several results for 2-clique-colorable graphs.

Results about colorings

Coloring graphs with few P_4's

- Many NP-hard problems were proved to be polynomial time solvable for cographs.

Results about colorings

Coloring graphs with few P_4's

- Many NP-hard problems were proved to be polynomial time solvable for cographs.
- Polynomial algorithms for acyclic and star colorings in cographs [Lyons, 2011].
- Polynomial algorithms for acyclic, star and harmonious colorings in $(q, q-4)$-graphs [Campos et al., 2011].

Some superclasses of cographs

$(q, q-4)$-graph

- Every set with $\leq q$ vertices induces $\leq q-4$ induced P_4's.
- Cographs $= (4, 0)$-graphs.
- P_4-sparse graphs $= (5, 1)$-graphs.

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P_4-tidy

- Every induced P_4 $u-v-x-y$ has at most one vertex z such that $\{u, v, x, y, z\}$ induces more than one P_4.

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P_4-laden

- Every set ≤ 6 vertices induces at most 2 P_4's or is a split graph.

Main theorems

Theorem 1

There exist linear time algoritms to obtain the Thue and the clique chromatic numbers of P_4-tidy and $(q, q-4)$-graphs, for every fixed q.

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Every acyclic coloring of a cograph is also nonrepetitive.

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Theorem 2

Every acyclic coloring of a cograph is also nonrepetitive.

Theorem 3

- Every P_4-tidy is 3-clique-colorable.
- Every P_4-laden is 2-clique-colorable.
- Every connected $(q, q-4)$-graph with $\geq q$ vertices is 2-clique-colorable.

$(q, q-4)$-graphs

Structural Theorem

A graph G is a $(q, q-4)$-graph if and only if exactly one of the following holds :

(a) G is the union or the join of two $(q, q-4)$-graphs;

(b) G is a spider (R, C, S) and $G[R]$ is a $(q, q-4)$-graph;

(c) G has $< q$ vertices or $V(G) = \emptyset$;

(d) G contains a subgraph H, with bipartition (H_1, H_2) and $|V(H)| < q$, $G - H$ is a $(q, q-4)$-graph and every vertex of $G - H$ is adjacent to every vertex of H_1 and non-adjacent to every vertex of H_2.

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(b) G is isomorphic to P_5, $\overline{P_5}$, C_5, K_1 or $V(G) = \emptyset$;

(c) G is a quasi-spider (R, C, S) and $G[R]$ is a P_4-tidy graph.

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A graph G is a P_4-laden graph if and only if exactly one of the following holds :

(a) G is the union or the join of two P_4-laden graphs;
(b) G is isomorphic to P_5, $\overline{P_5}$, K_1, $V(G) = \emptyset$ or a split graph;
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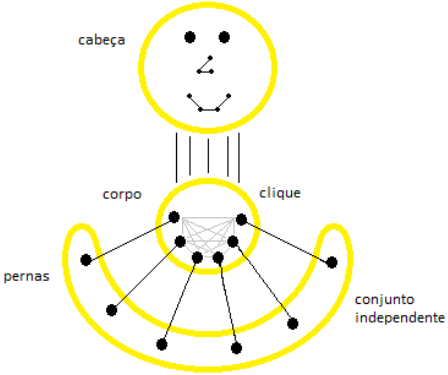


Figure: A spider

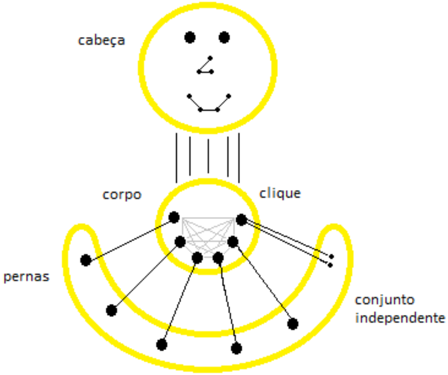
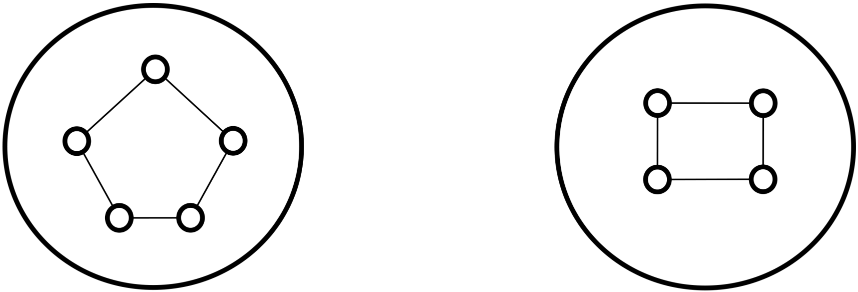


Figure: A quasi-spider

Non-repetitive coloring for unions and joins

Lemma

Given graphs G_1 and G_2 with n_1 and n_2 vertices respectively :
$\pi(G_1 \cup G_2) = \max\{\pi(G_1), \pi(G_2)\}$,
$\pi(G_1 \vee G_2) = \min\{\pi(G_1) + n_2, \pi(G_2) + n_1\}$.



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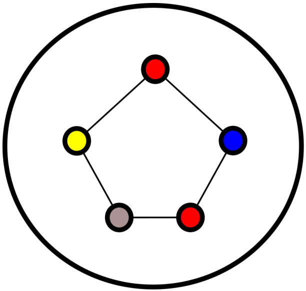

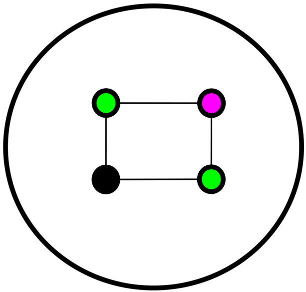

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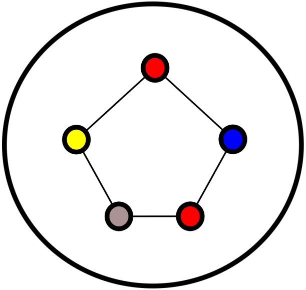

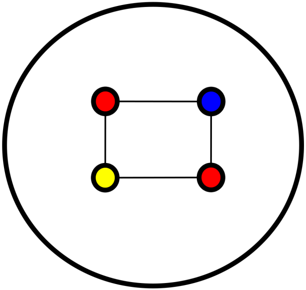

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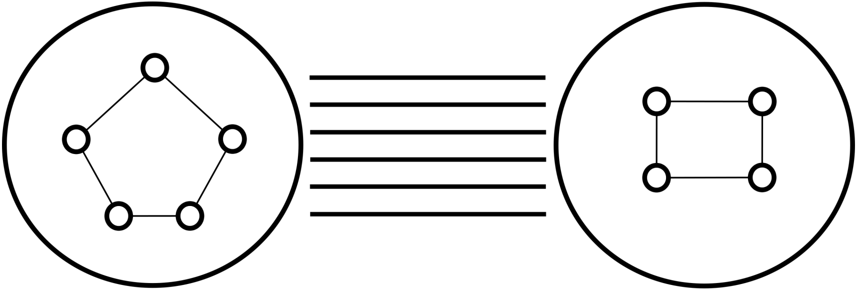

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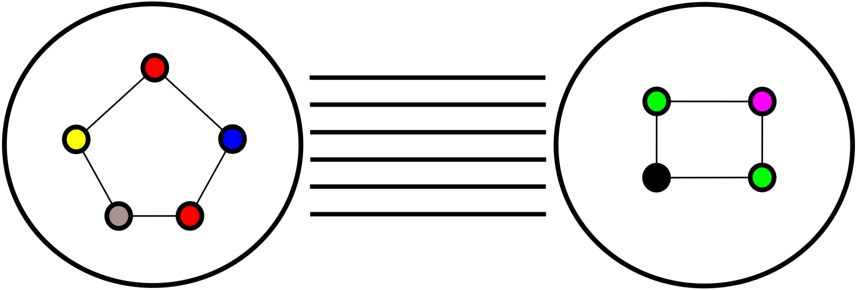

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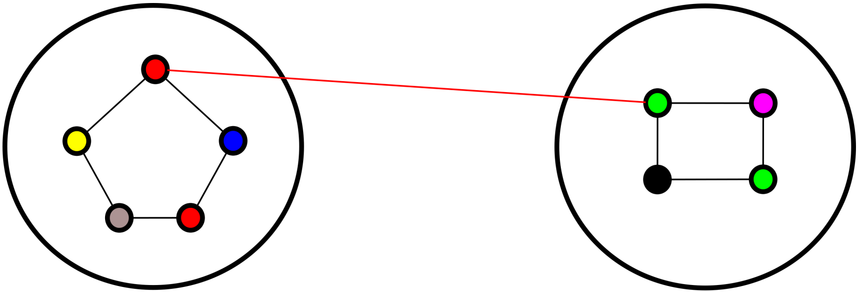

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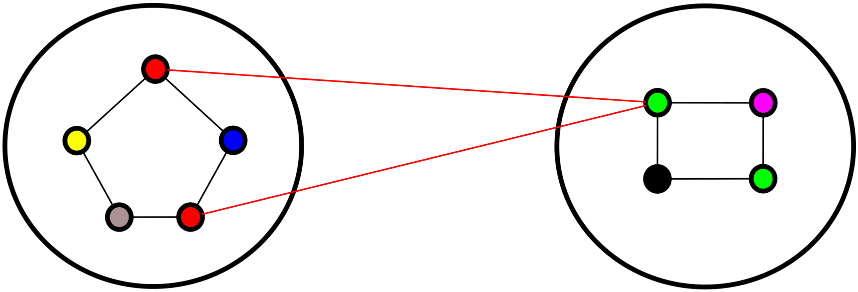

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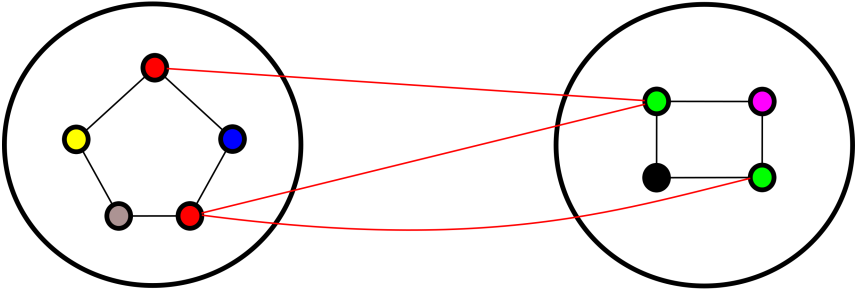

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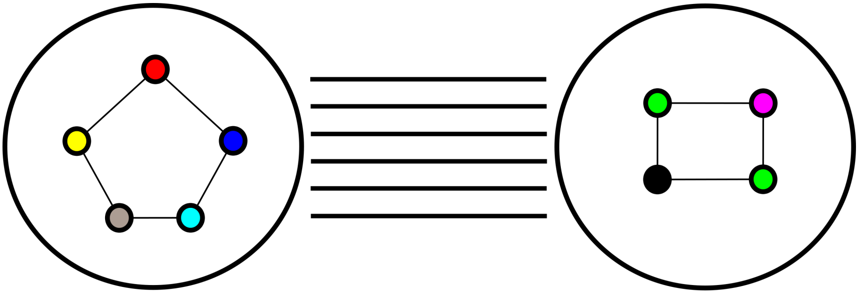


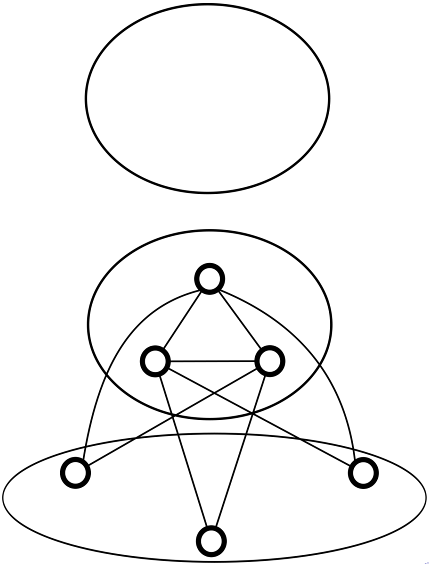
Non-repetitive coloring for spiders

Lemma

Let G be a spider (R, C, S), where $|C| = |S| = k$. Then

$$\pi(G) = \begin{cases} k+1, & \text{if } R = \emptyset \text{ and } G \text{ is thick} \\ \pi(G[R]) + k, & \text{otherwise.} \end{cases}$$

Non-repetitive coloring for spiders



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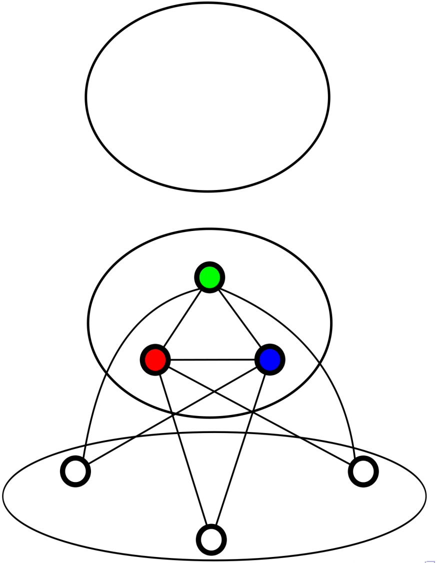

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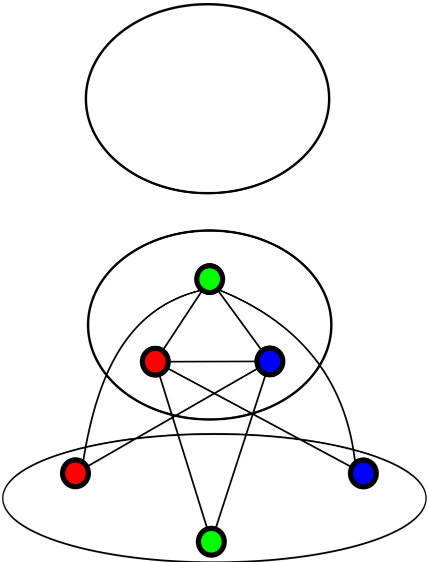

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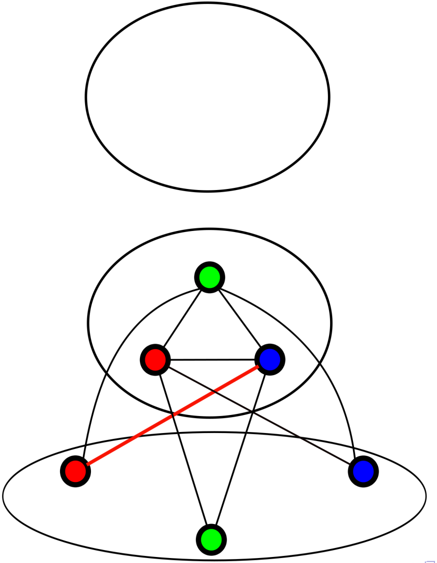

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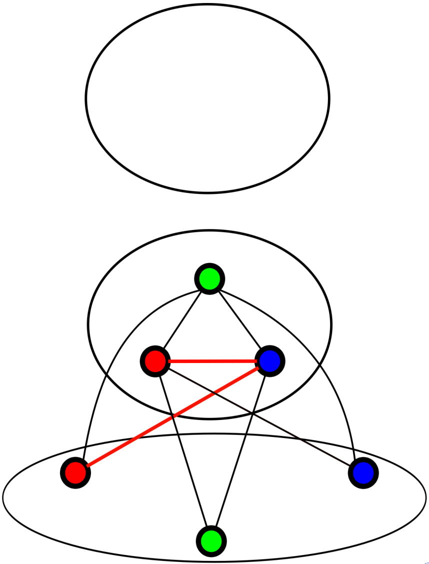

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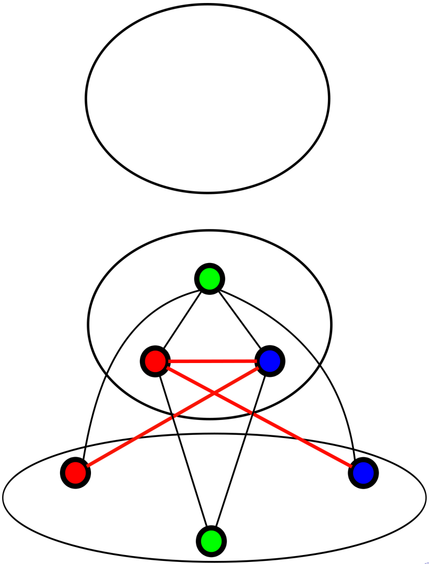

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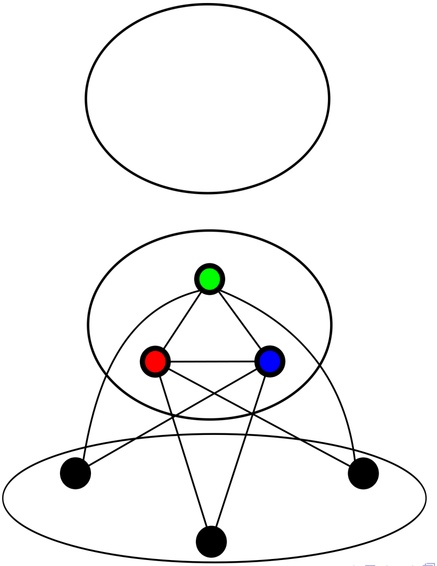

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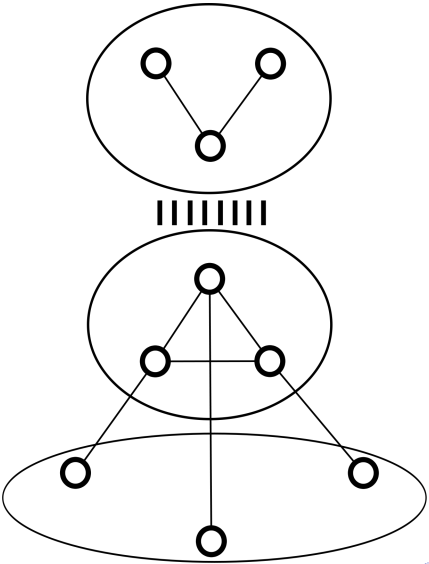

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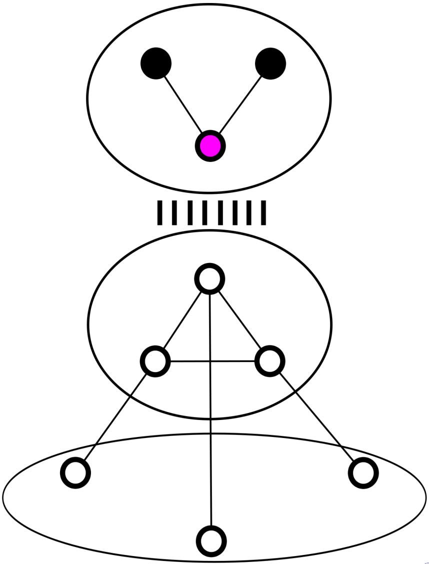

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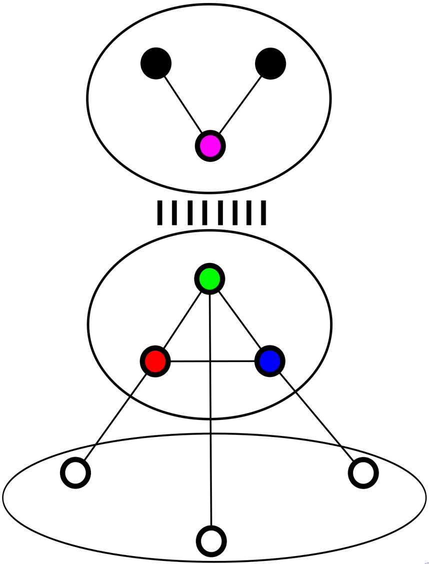

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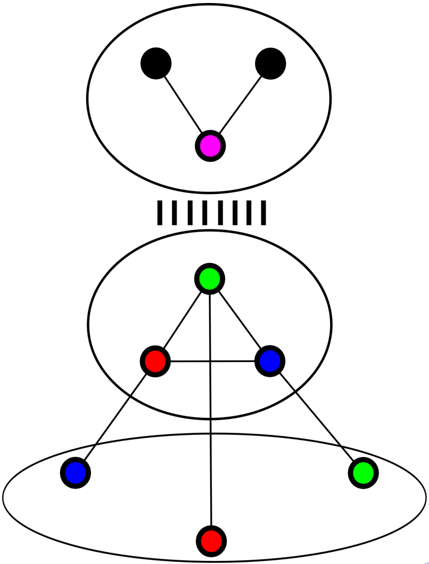

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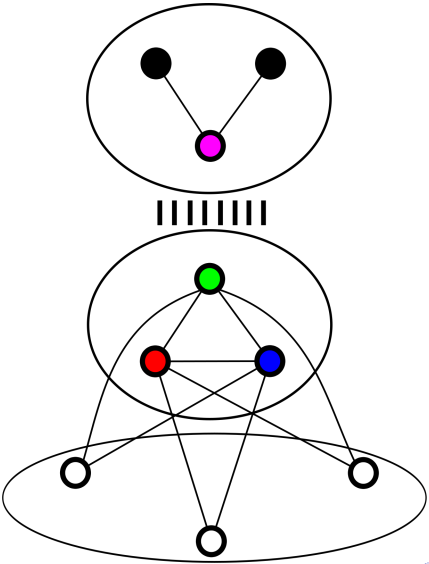

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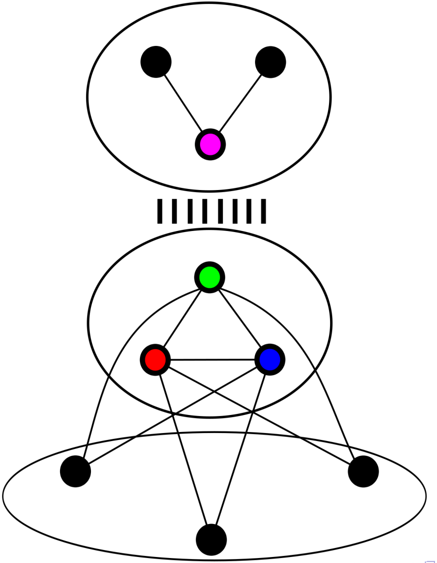

Non-repetitive coloring for quasi-spiders

Lemma

Let G be a quasi-spider (R, C, S) such that $\min\{|C|, |S|\} = k > 3$ and $\max\{|C|, |S|\} = k + 1$. Let $H = K_2$ or $H = \overline{K_2}$ be the subgraph that replaced a vertex of $C \cup S$. Then

$$\pi(G) = \begin{cases} \pi(G[R]) + k, & \text{if } H \in S \text{ and } G \text{ is thin,} \\ \pi(G[R]) + k, & \text{if } H \in S,\ G \text{ is thick} \\ & \qquad\qquad \text{and } R \neq \emptyset, \\ \pi(G[R]) + k + 2, & \text{if } H \in C,\ G \text{ is thick} \\ & \qquad\qquad \text{and } R = \emptyset, \\ \pi(G[R]) + k + 1, & \text{otherwise.} \end{cases}$$

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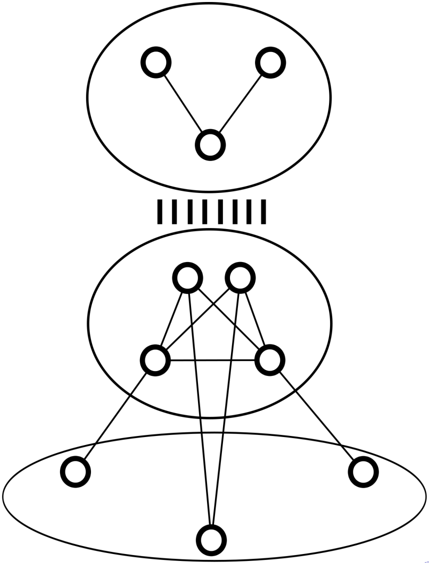

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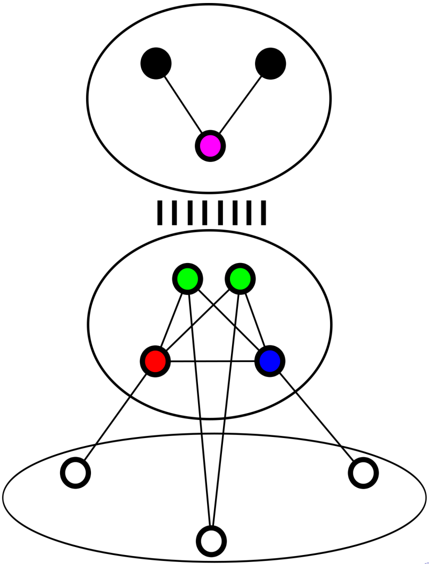

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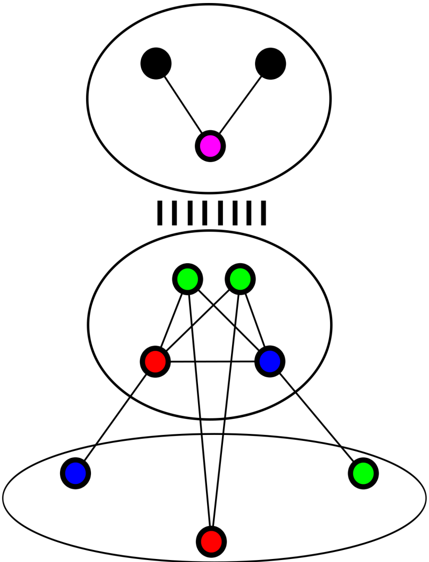

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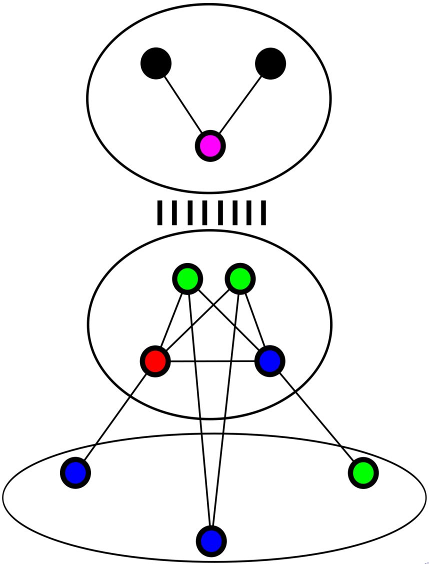

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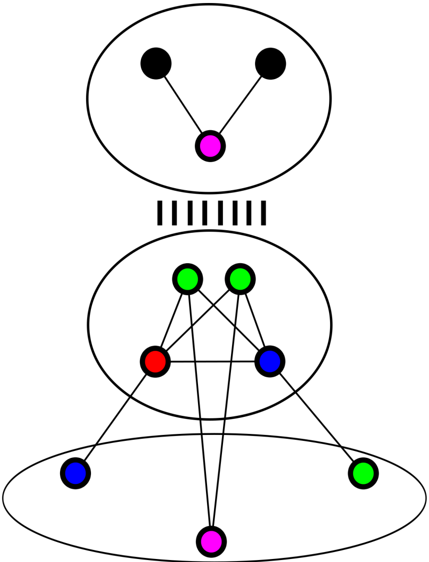

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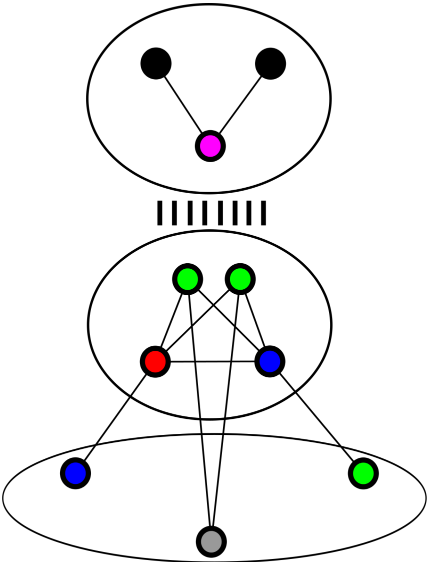

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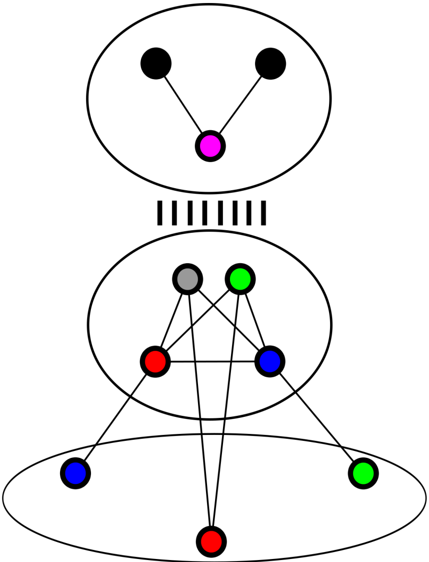

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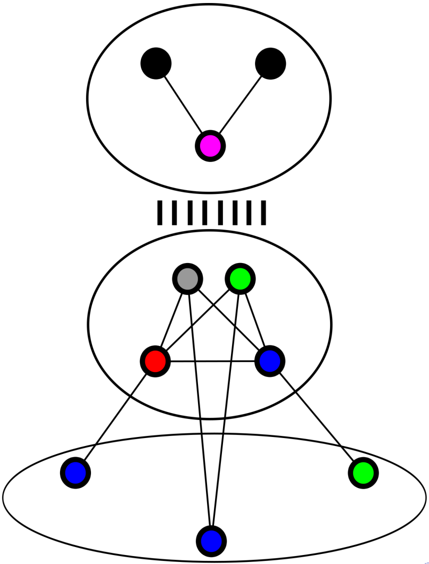

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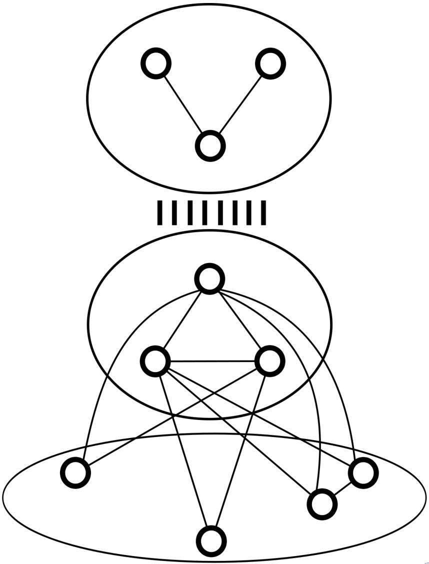

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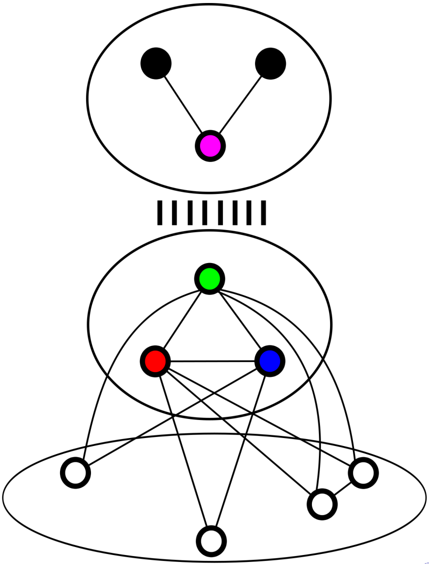

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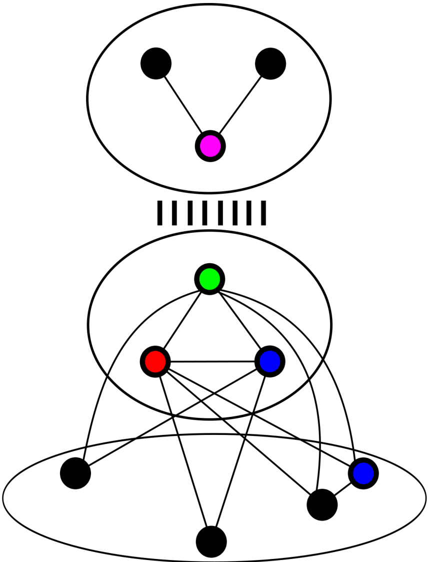

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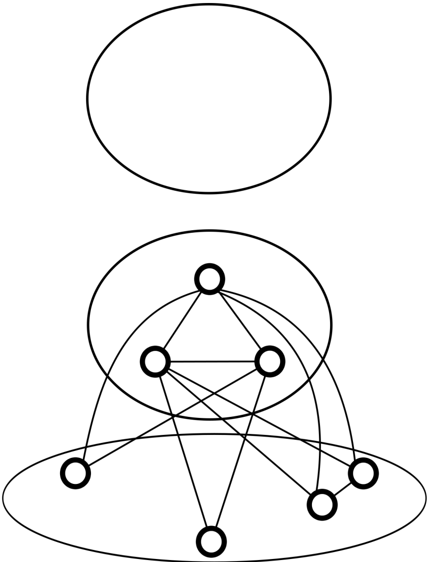

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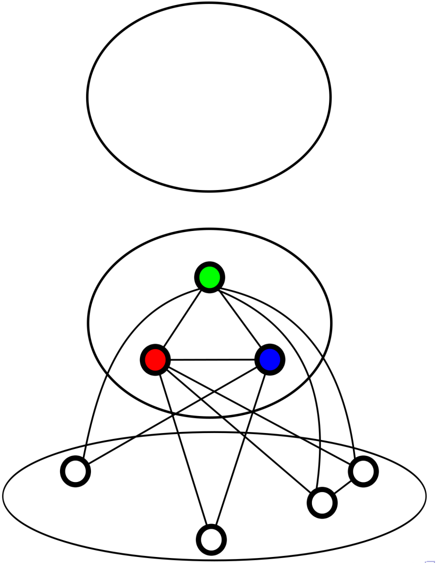

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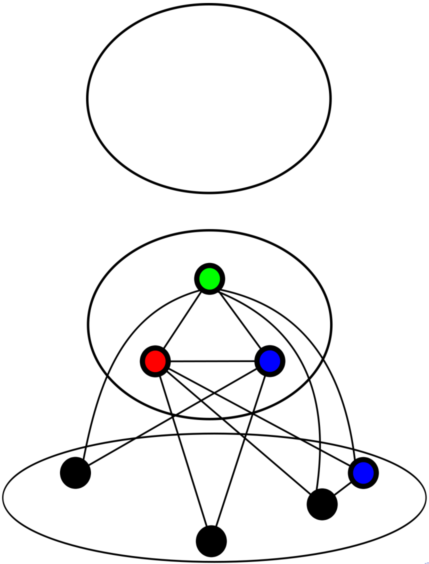

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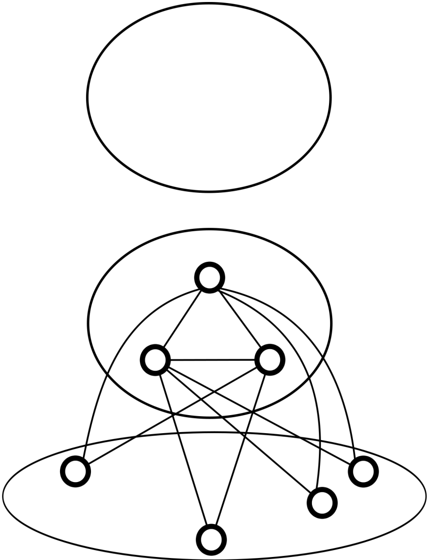

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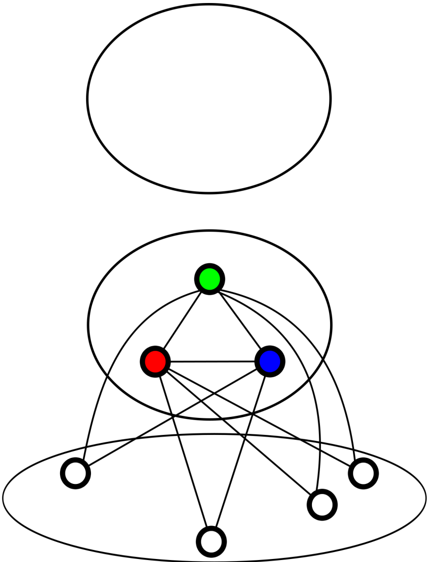

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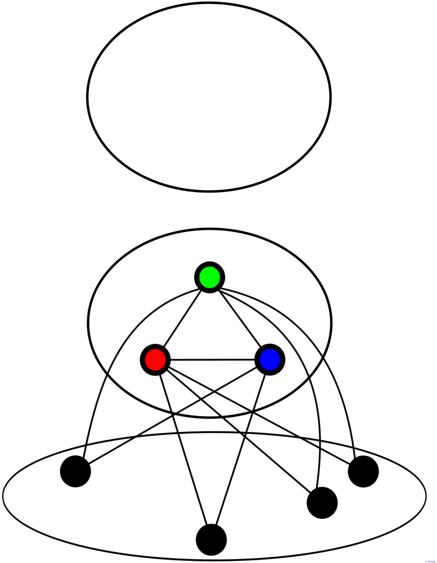

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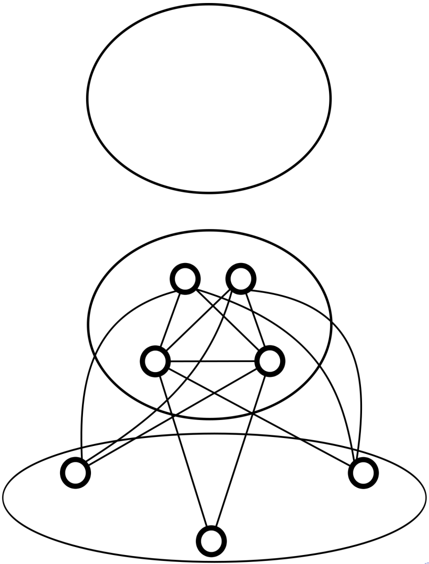

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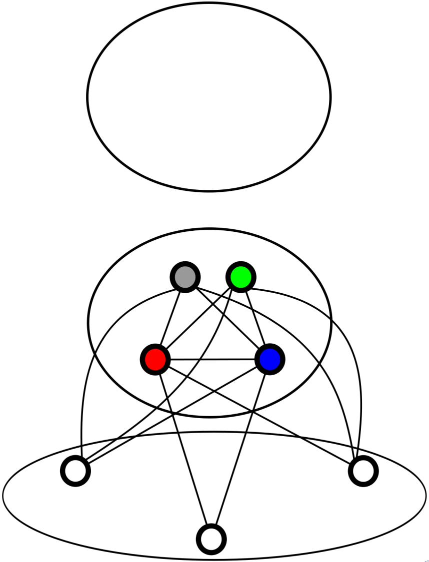

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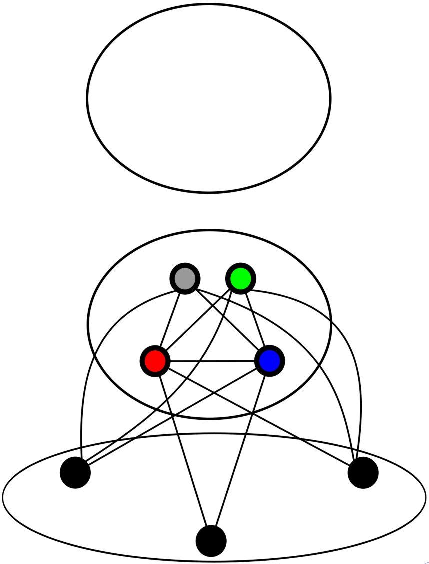

Clique coloring for unions, joins and quasi-spiders

Lemma

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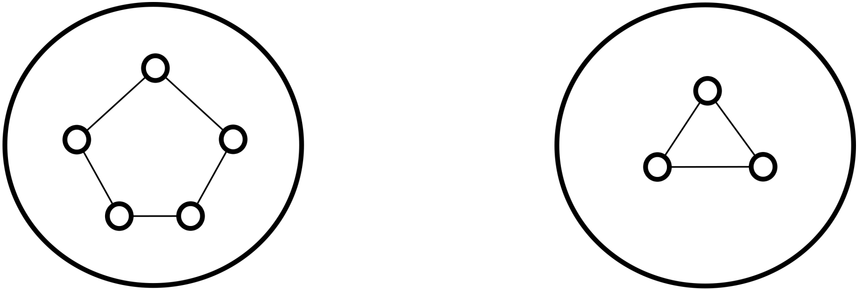

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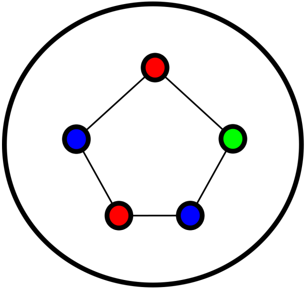

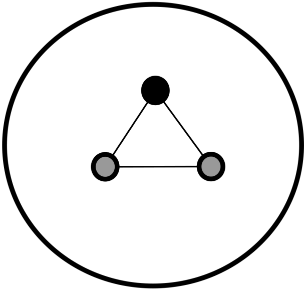

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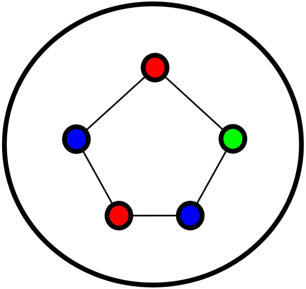

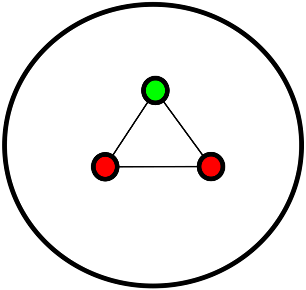

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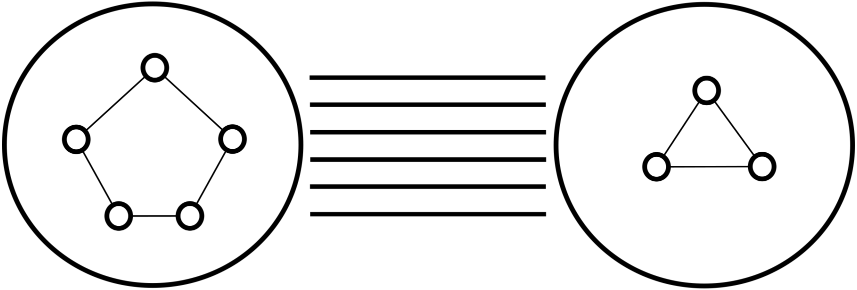

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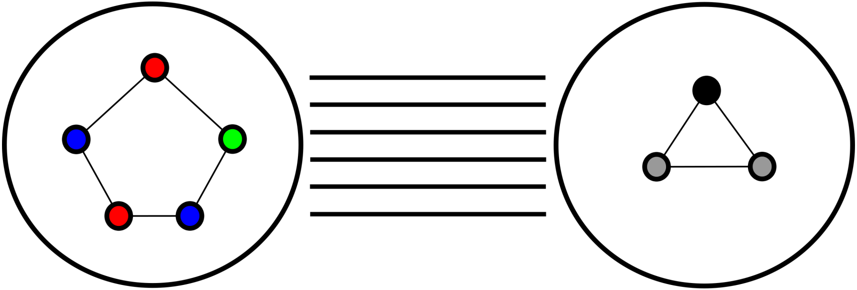

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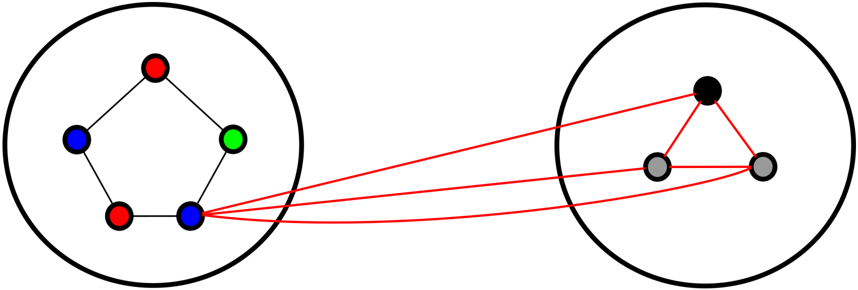

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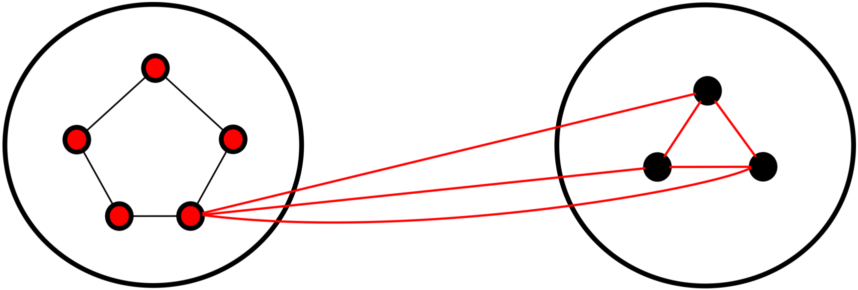

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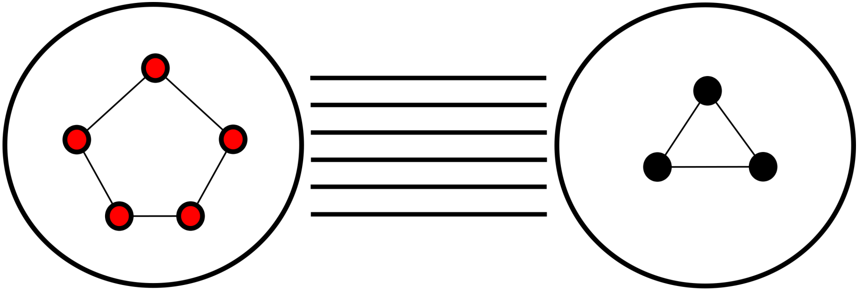

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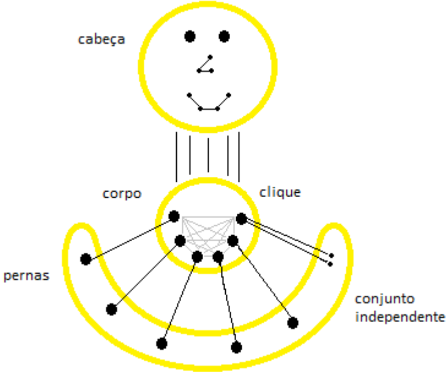

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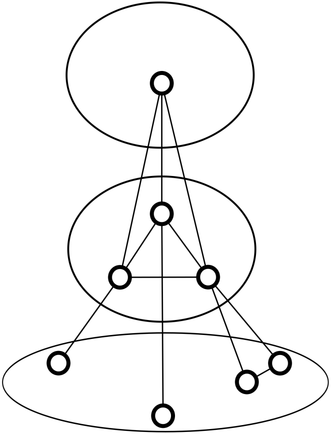

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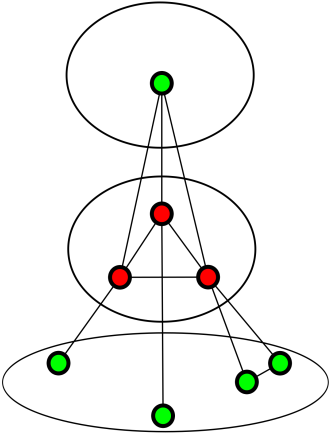

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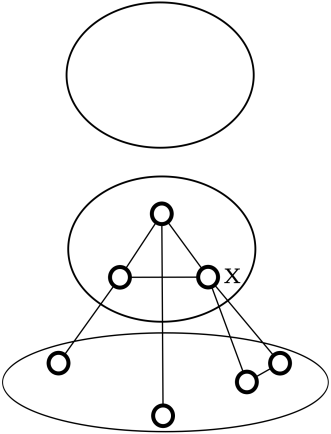

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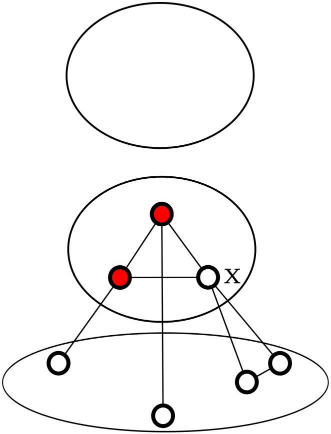

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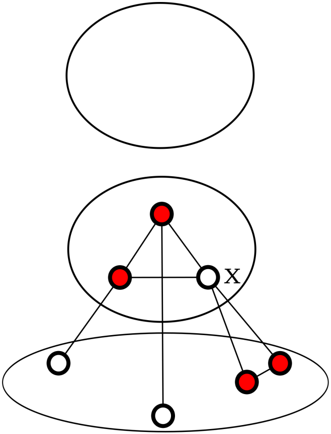

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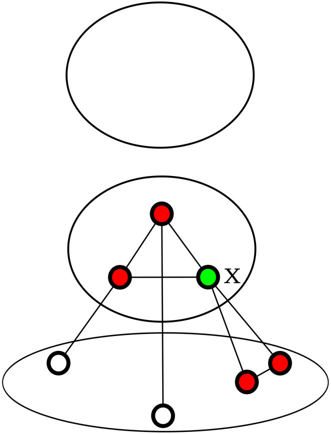

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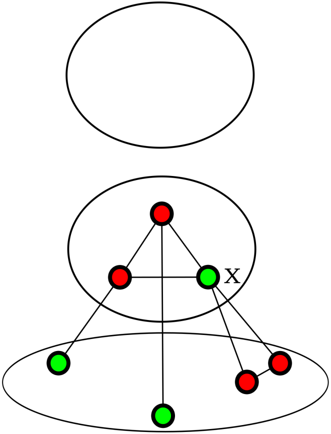

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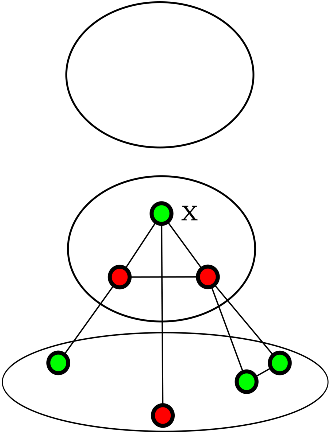


Coloring (q,q-4)-graphs

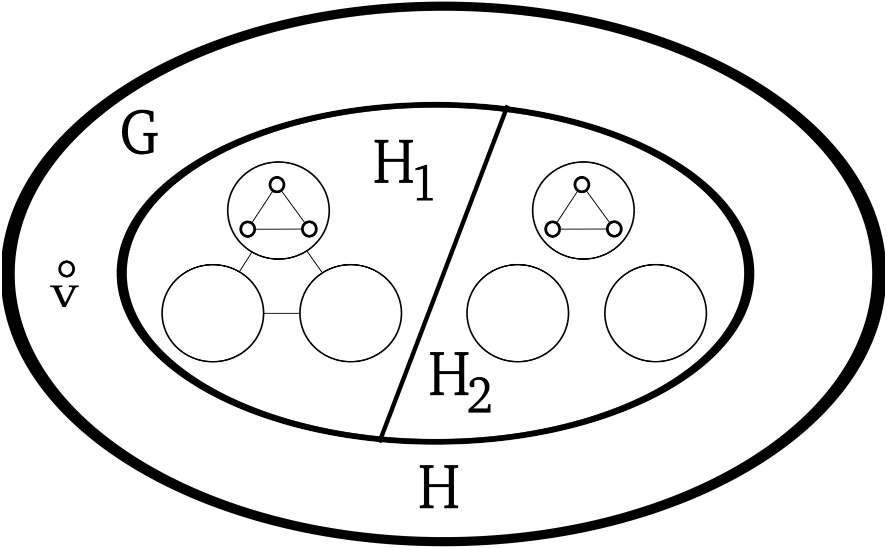
Lemma

If $G - H$ is not empty, then $\chi_c(G) = 2$ (coloring the vertices of $G - H$ and H_2 with the color 1 and the vertices of H_1 with the color 2). If $G - H$ is empty, then G has less than q vertices and

$$\chi_c(G) = \min_{\psi \in C_c(H)} \Big\{ k(\psi) \Big\},$$

where $C_c(H)$ is the set of all clique-colorings of H.

Coloring (q,q-4)-graphs



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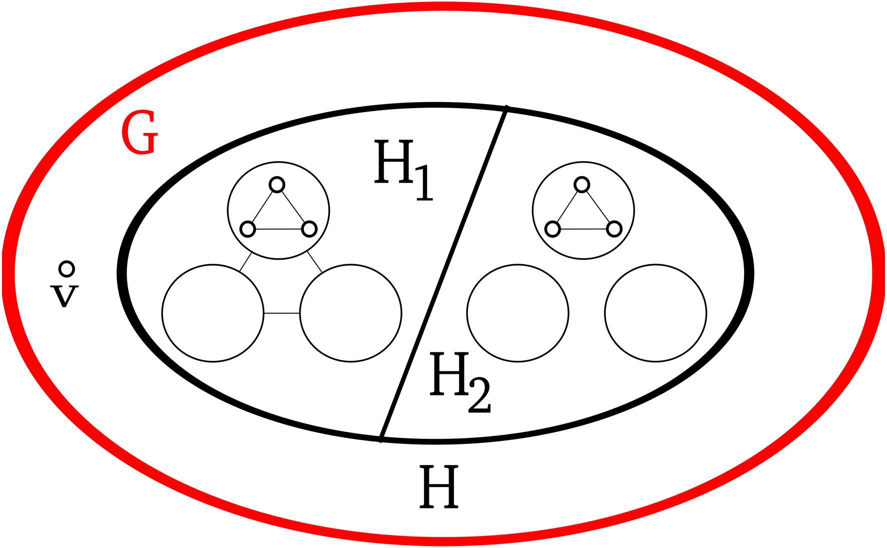

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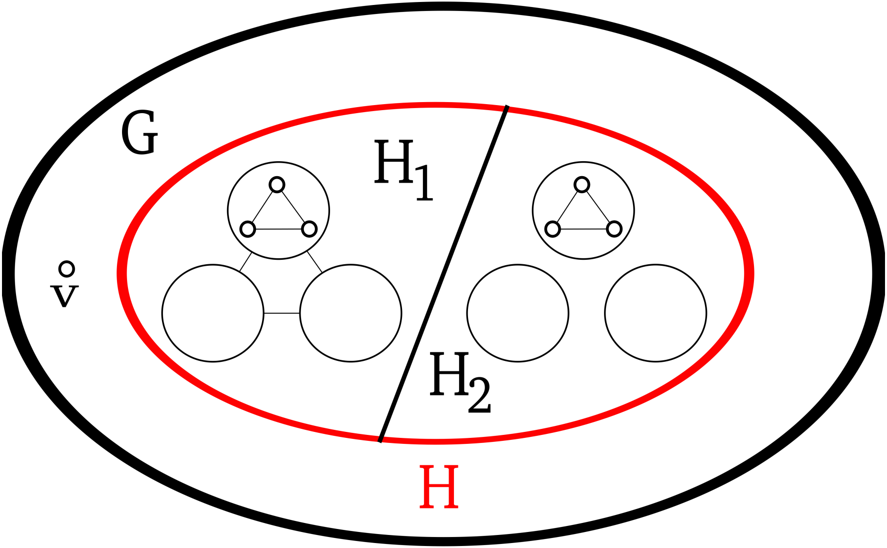

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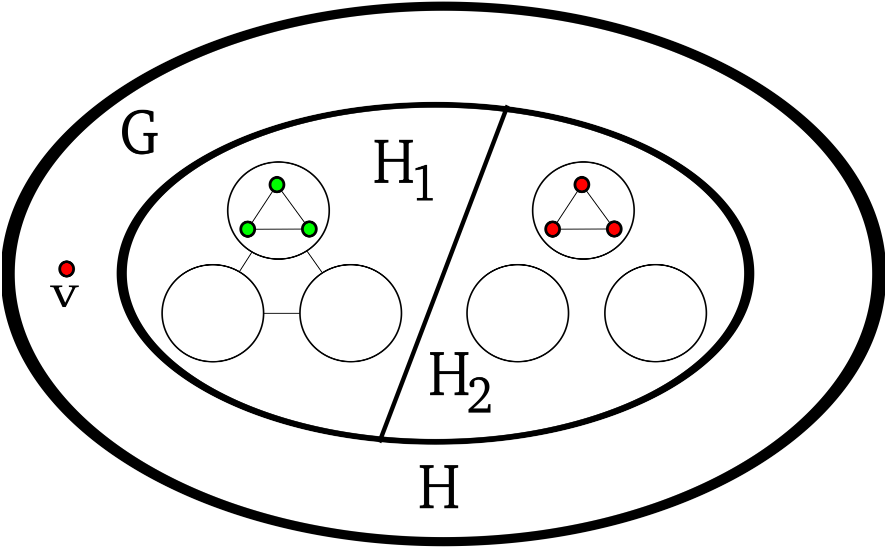

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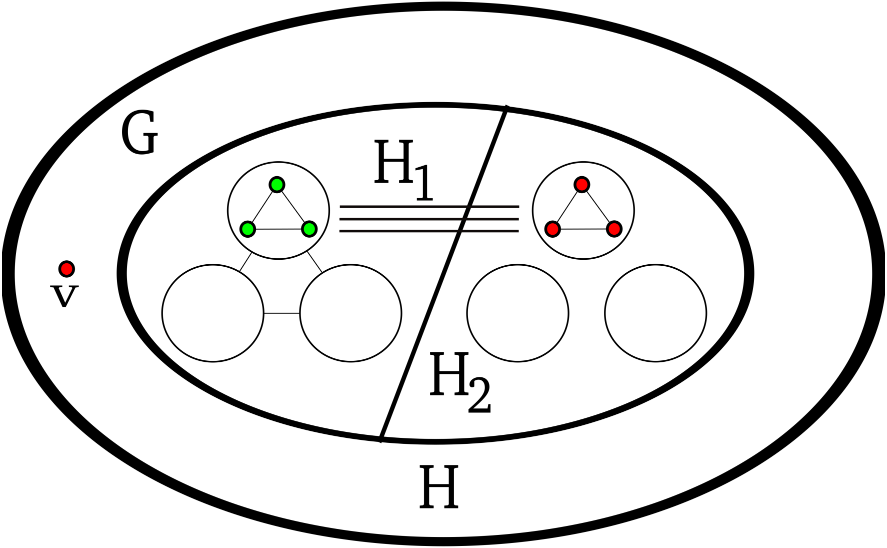

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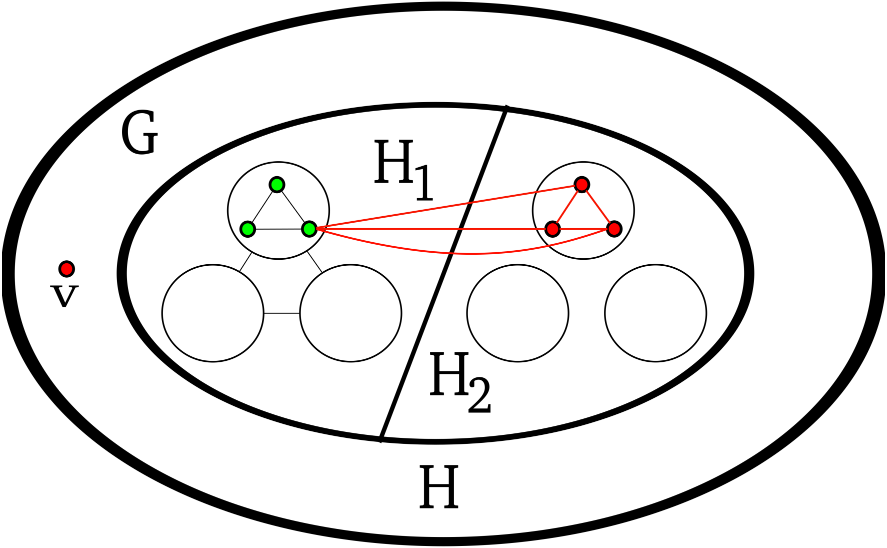

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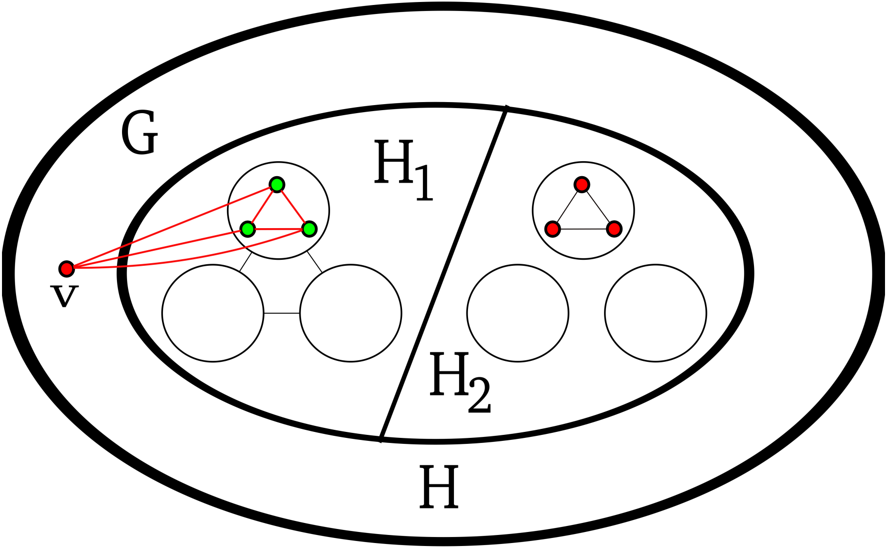

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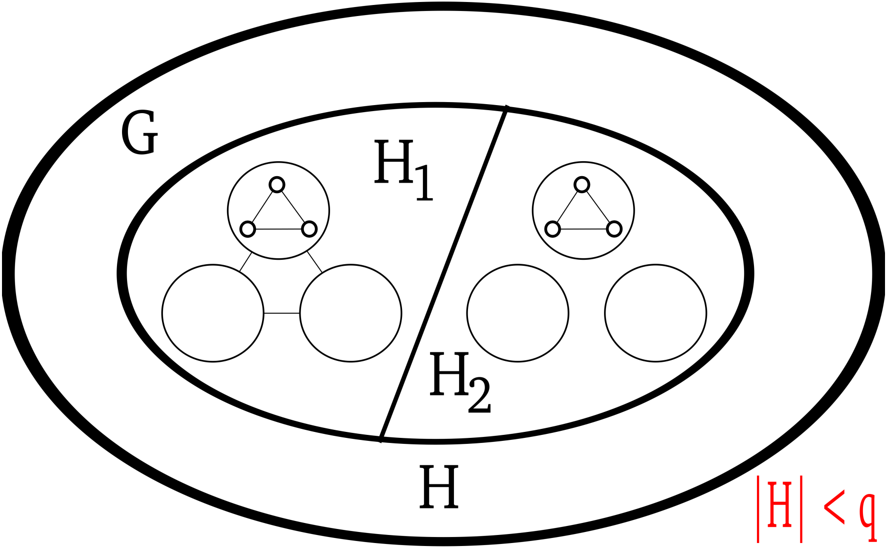

$|H| < q$

Coloring (q,q-4)-graphs

Lemma

Given a coloring ψ of H, let $k_2(\psi)$ be the number of colors with no vertex of H_1 and with no vertex of H_2 which is neighbor of two vertices from H_1 with the same color. Then

$$\pi(G) = \min\left\{ \min_{\psi \in C_\pi(H)} \left\{ k(\psi) + \max\{0, n' - k_2(\psi)\} \right\}, \min_{\psi' \in C'_\pi(H)} \left\{ k(\psi') + \max\{0, \pi(G-H) - k_2(\psi')\} \right\} \right\}$$

Main theorems

Theorem 1

There exist linear time algoritms to obtain the Thue and the clique chromatic numbers of P_4-tidy and $(q, q-4)$-graphs, for every fixed q.

Theorem 2

Every acyclic coloring of a cograph is also nonrepetitive.

Theorem 3

- Every P_4-tidy is 3-clique-colorable.
- Every P_4-laden is 2-clique-colorable.
- Every connected $(q, q-4)$-graph with $\geq q$ vertices is 2-clique-colorable.

Main results

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If G is P_4-tidy, then the clique chromatic number is at most 3.

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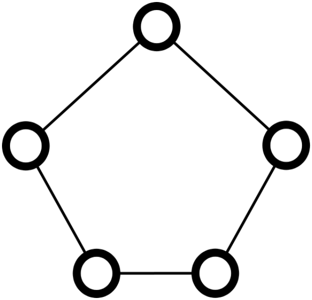

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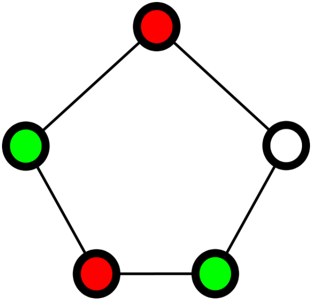

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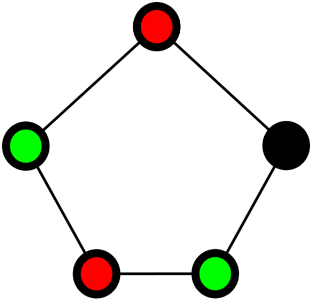

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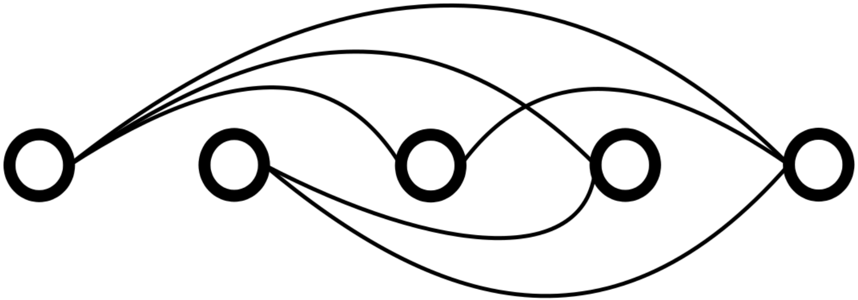

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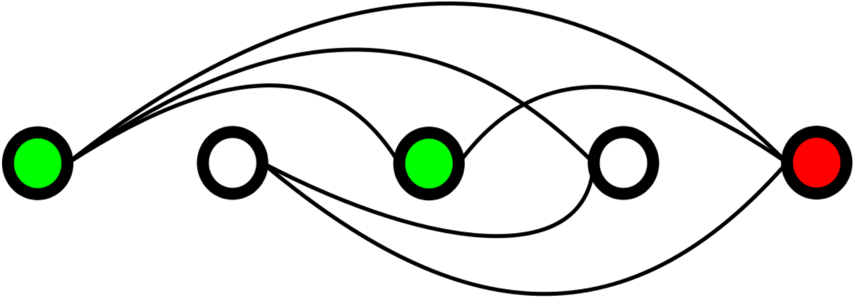

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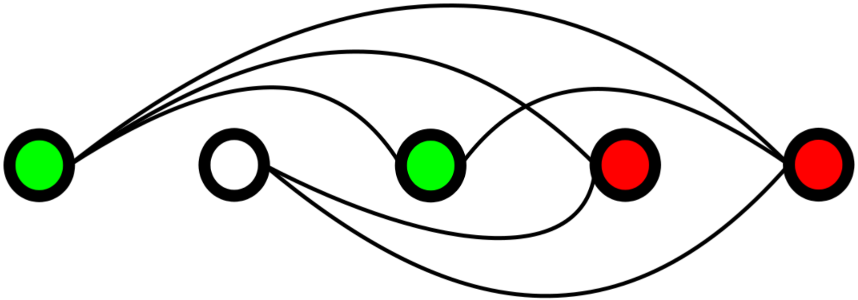
If G is P_4-laden, then the clique chromatic number is at most 2.



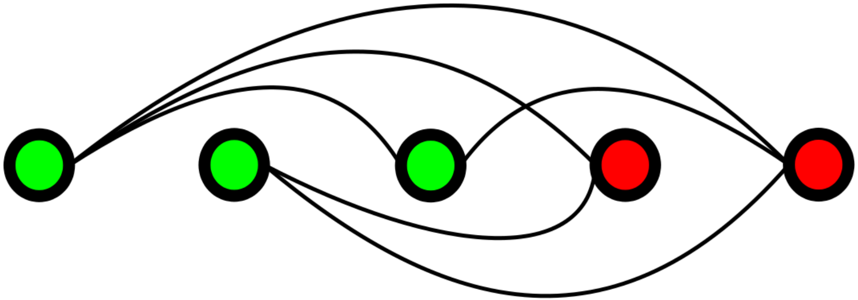
Main results

Lemma

If G is P_4-tidy, then the clique chromatic number is at most 3.

Lemma

If G is P_4-laden, then the clique chromatic number is at most 2.

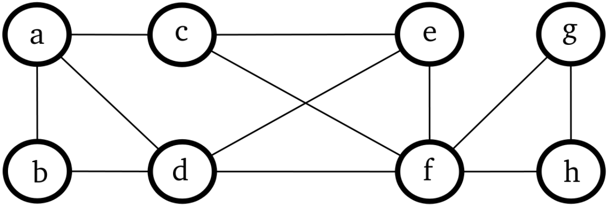


Split decomposition

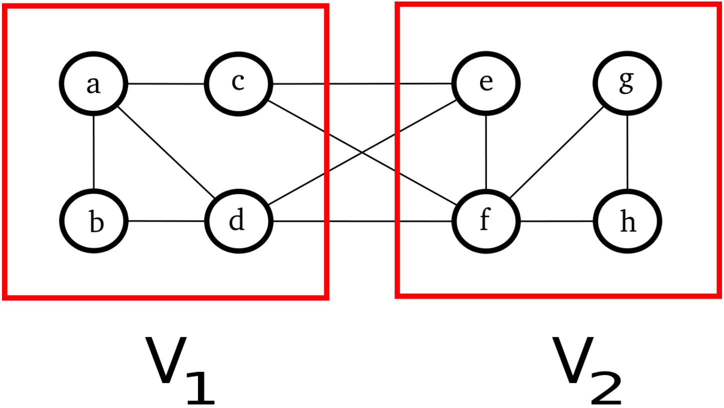
Theorem 4

There exist polynomial time algorithms to obtain an optimal acyclic coloring of distance hereditary graphs and graphs with a given split decomposition with bounded width.

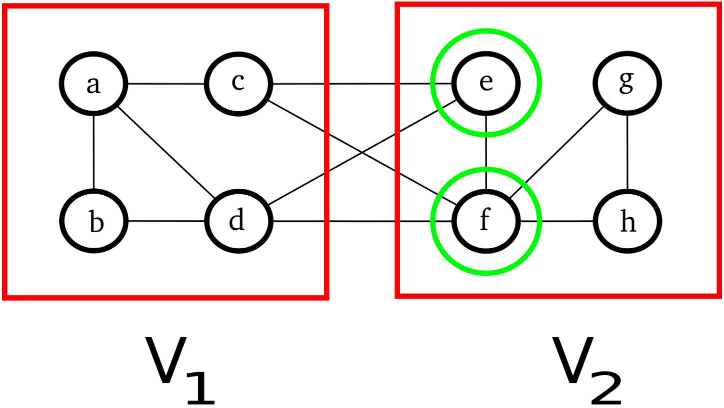
Split decomposition



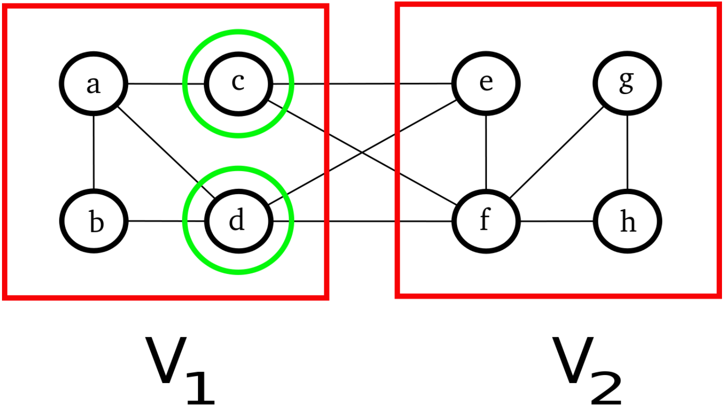
Split decomposition



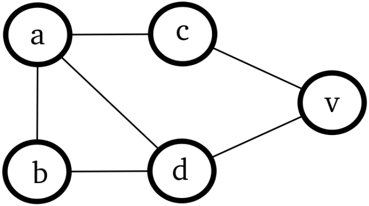
Split decomposition

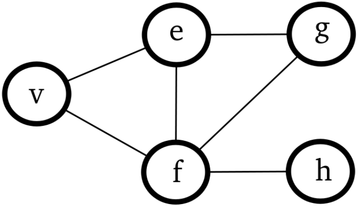


Split decomposition

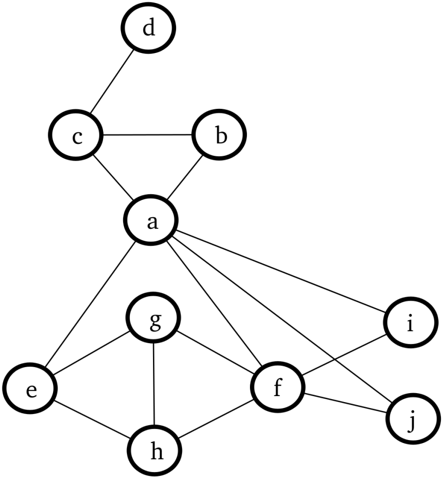


Split decomposition

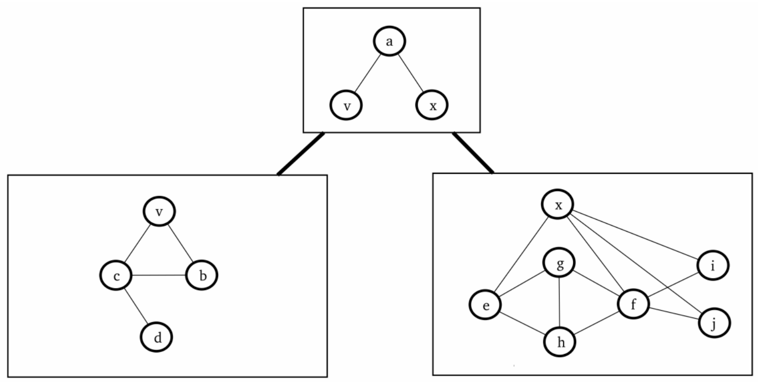




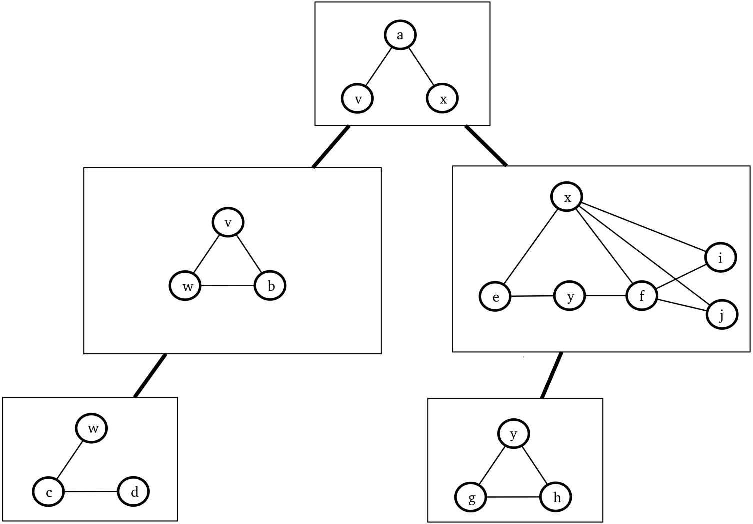
Split decomposition



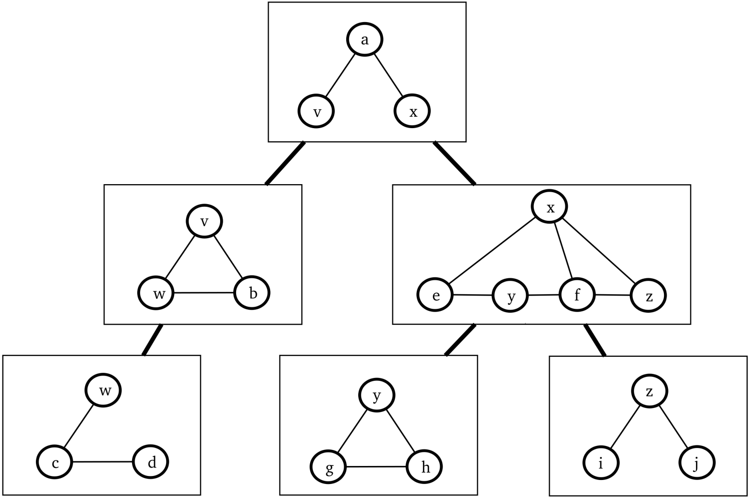
Split decomposition



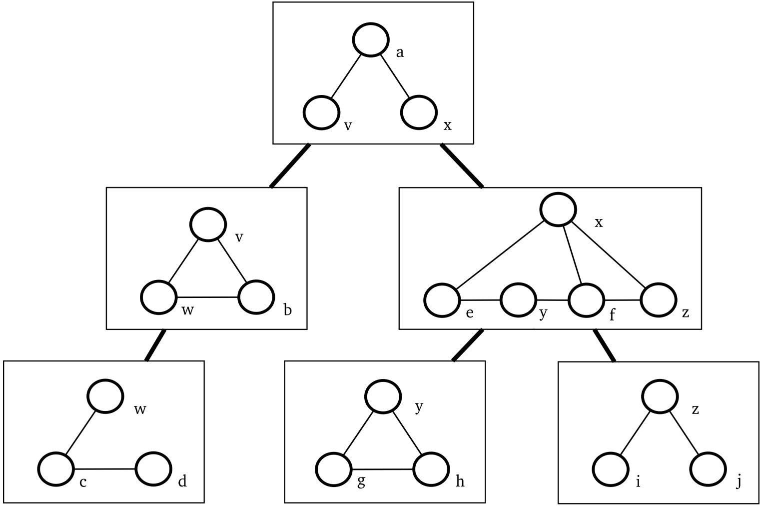
Split decomposition



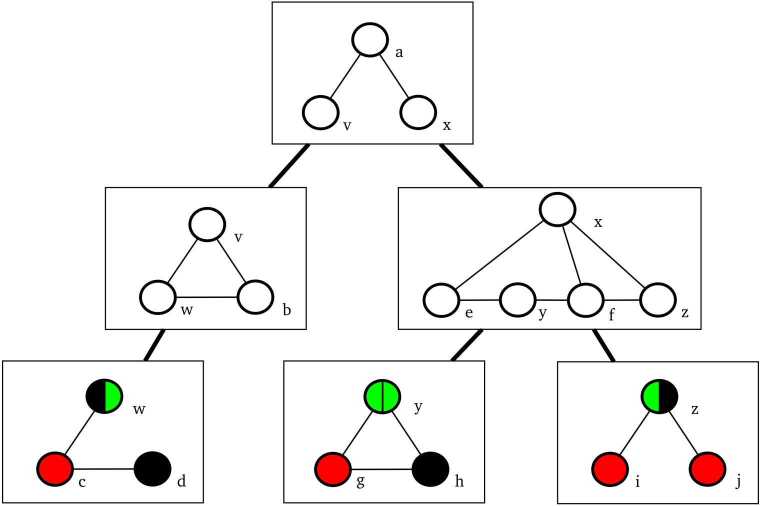
Split decomposition



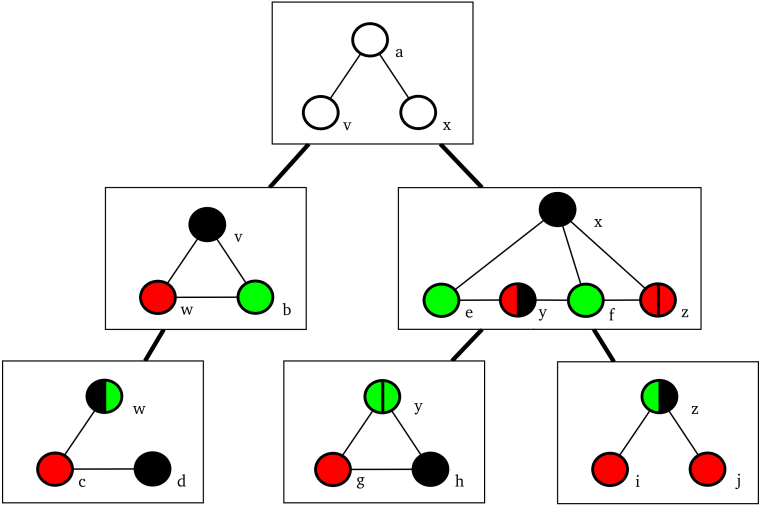
Split decomposition (coloring : [M. Rao, 2008])



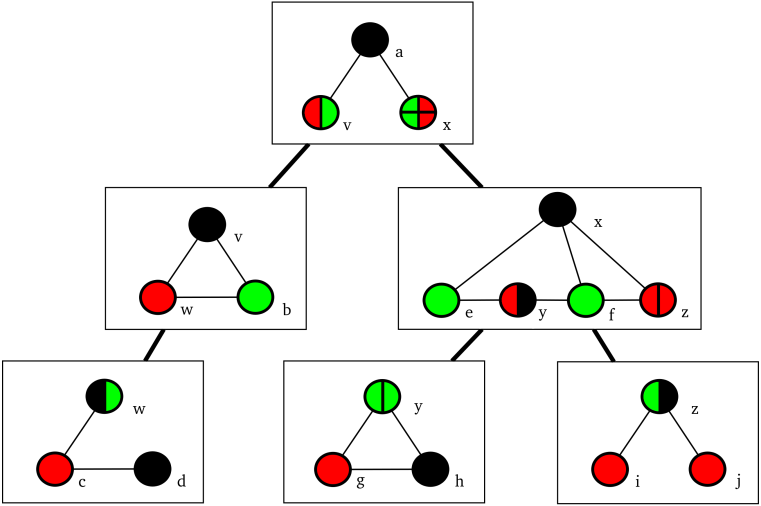
Split decomposition (coloring : [M. Rao, 2008])



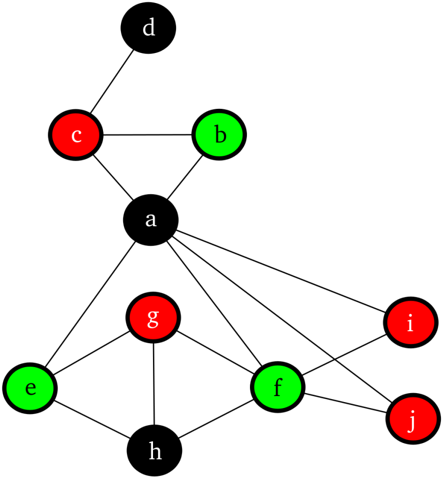
Split decomposition (coloring : [M. Rao, 2008])



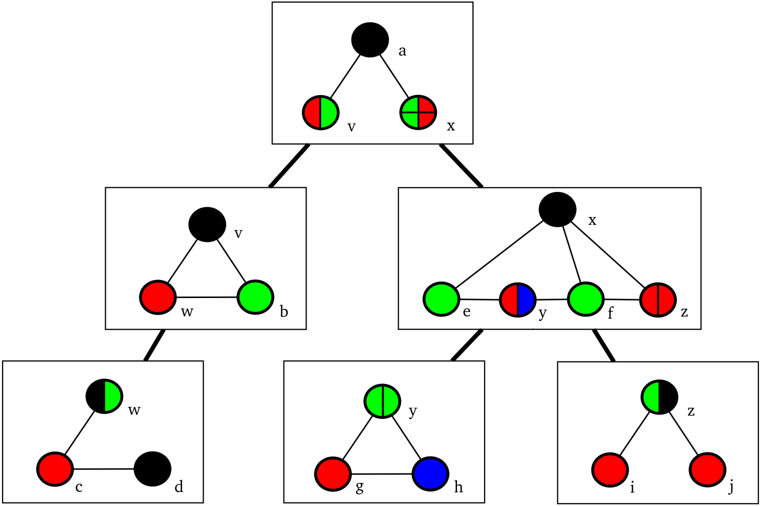
Split decomposition (coloring : [M. Rao, 2008])



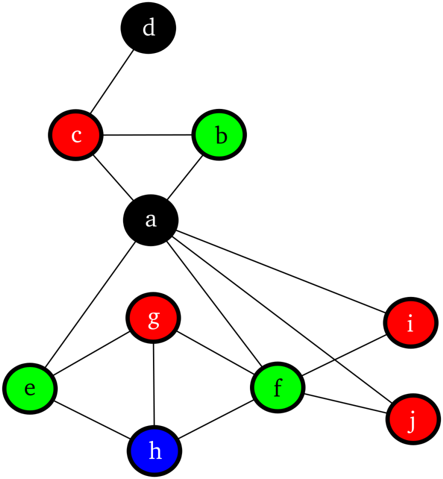
Split decomposition (coloring : [M. Rao, 2008])

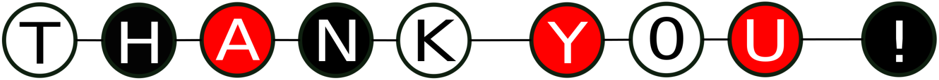


Split decomposition (acyclic coloring)



Split decomposition (acyclic coloring)





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Nonrepetitive, acyclic and clique colorings of graphs with few P_4's

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26 of Setember of 2012 (15:20 - 15:45)
CLAIO/SBPO 2012 (Rio de Janeiro, RJ)